Importance of Syndication Contracts in Venture Capital Matching Markets

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Abstract

There are a multitude of reasons for VC syndication documented in the literature, from capital constraints to risk mitigation. The superior performance of portfolio companies receiving syndicated investments from VCs has also been explored. However, the systematic selection into syndicates and the performance of syndicated investments have not been modeled jointly, despite the fact the two are inextricably linked. I estimate a multi-index sample selection model in order to examine how syndication of venture capital investments affects portfolio company outcomes. I find that VCs and portfolio companies use syndication contracts as a selection mechanism, which leads to sorting into syndicates along both observable and unobservable characteristics. Furthermore, I determine that changes in sorting along unobservables over time results in the standard Probit model overestimating the value-added from syndication in some years, while underestimating it in others. Finally, I illustrate the negative impact of increases in the long-term capital gains tax rate driven by VC and company exits, as well as contract switching.

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1. Introduction

After the bursting of the Dotcom bubble and the subsequent low point in venture capital investment during 2002, the venture capital industry has been on an upward trajectory. The only substantial dip was during the first year of the Great Recession. As a result of this sustained growth, 2018 and 2019 recorded levels of investment similar to the height of the Dotcom bubble, at around $118.3 and $107.8 billion USD respectively. The robust performance of the industry is one of the many reasons it has garnered so much attention from researchers. Furthermore, unlike most other sectors of the finance industry, the direct outcome of venture capital investments is new job creation. As a result, the venture capital industry is viewed as vital to the growth and well-being of the economy. Kortum and Lerner (2000) find that the venture capital industry was responsible for 8% of industrial innovations from 1983 to 1992, which seems trivial at first glance. However, in 1992, US venture capital investments represented only .063% of GDP. Thus, the industry not only offers job growth, but is also comprised of companies engaging in highly innovative activities. One feature of the VC industry is the syndication of investments by VCs. Syndication broadly refers to the process through which a VC invites other VCs to invest in a company jointly (i.e. syndicate formation).\footnote{Note, I will use entrepreneur and company interchangeably throughout the paper.} While the syndication of investments is not unique to the VC industry, loan syndication in the commercial and investment banking industries are just a couple of examples, it is far more impactful in the market for venture capital. From the beginning of 2010 until the end of 2019, 70.8% of investments in all rounds were syndicated. Of the $531.2 billion invested during that period, 80.5% of the money spent came from syndicated investments.\footnote{The monetary investment is measured in 2012 USD.} Thus, syndication is not only prevalent in the industry, but also accounts for a substantial percentage of total investment. Pinterest, Inc. is an example of a well known company that received a syndicated investment led by Andreessen Horowitz LLC, which became a so-called tech "unicorn" en route to a 2019 IPO. While the syndicated match between Pinterest, Inc. and lead VC Andreessen Horowitz LLC ended with success, this is obviously not always the case. The once promising biotech startup, Sequel Pharmaceuticals, received the largest initial investment in 2007 from a syndicate comprised of six VCs led by Domain Associates LLC, but received its last infusion of cash in 2010 before becoming defunct. Although the numbers provide a tangible measurement of the importance of syndication to the VC industry, it does not elucidate the reader as to why it is important.

The formation of VC syndicates has been of particular interest to researchers from a variety of business related backgrounds. The heavy involvement of VCs in daily company operations opens an avenue for additional investors to influence the company outcome beyond the simple outlay of capital that is the hallmark of all other syndicated investment, see Bottazzi et al. (2008). Second, companies receive multiple rounds of investment, with each round exhibiting its own distinctive syndication patterns. Earlier rounds are plagued by uncertainty, with syndication used to mitigate risk; while later rounds tend to involve more profitable companies, which attracts additional VCs hoping to take advantage of an all but riskless investment. Third, there is a vast amount of heterogeneity in both the entrepreneurs seeking capital, and the VCs selecting investments. Companies can be separated into a wide range of industries, some of which require highly specialized knowledge to navigate. These reasons illustrate the complex nature of syndication in the VC industry which make it an appealing research subject.

The decision to syndicate an investment is multifaceted. Of particular interest, in this much studied literature, is discerning the relative importance of the different aspects affecting the decision. Among these components, three tend to receive the most focus, they are: (i) value-added, (ii) selection, and (iii) finance-related. The first refers to the value added
to the company post-investment due to the additional advisory and monitoring capabilities that co-investing VCs can provide. The second alludes to the pooling of knowledge by VCs to improve screening of company quality prior to an investment; known commonly as the selection effect. The third is the motivation to syndicate due to financial constraints or to spread risk. The value-added effect, which I refer to as VC VA, has been found to be more influential in the decision-making process than the benefit from selection (Brander et al., 2002; Manigart et al., 2006; Hochberg et al., 2007; Wang et al., 2012). On the other hand, (Wright and Lockett, 2003; Manigart et al., 2006) find that the finance-related motive is the most important driver of the syndication decision. Additionally, Manigart et al. (2006) show that the relative importance of these three components is different across VCs. The finance-related motive is less important for larger, less financially constrained VCs; while the VC VA and selection motives are more important for investments in early stage companies. Conversely, Lockett and Wright (2001), determines that risk diversification is an influential motivator for syndication in all rounds of company investment, but the VC VA is more important for early stage investments. Given the varying degree of importance for these factors across different investment stages, it is unsurprising that some papers have found conflicting results when examining the relationship between the motivation for syndication and other VC industry variables. However, the literature largely examines these three main components of syndicate formation separately, when they are necessarily interdependent. The need to account for these motives simultaneously is especially important when attempting to determine the impact of syndication on the likelihood a company achieves an IPO.

There are two main issues facing the econometrician when attempting to identify the impact of syndicate formation on company outcomes. One source of endogeneity comes from the VCs and companies systematically selecting into syndicates along unobserved characteristics. For example, a partner at a potential co-investing VC firm may have a relationship with the company CEO. This causes the error term in the outcome equation to become correlated with syndication. The other problem arises from a simultaneous causality bias due to the syndication of an investment being co-determined with the match. Unlike other characteristics available to the econometrician, such as VC experience, syndication is a match-specific variable the value of which is observed only in the event that a VC and company match. Thus, syndication is a function of the matching market equilibrium. On the other hand, given the motives previously discussed, syndication can be beneficial to both sides of the market. As a result, the equilibrium matching is influenced by syndication, which leads to the simultaneity bias. There are multiple approaches to dealing with the interrelated components guiding the syndication decision. Das et al. (2011) address the issue by including first stage probit predicted probabilities of syndication for linear outcomes, and estimating a biprobit for binary outcomes. They conclude that the selection effect from syndication results in higher returns, and that the value-added from syndication increases the probability of a positive company outcome. However, not only is finding good instruments for endogenous variables in the VC industry notoriously difficult, but, as Sørensen (2007) shows, sorting along unobserved VC and company characteristics leads to a substantial overestimation of the value-added from VC experience. Consequently, both results from Das et al. (2011) are likely to be biased. The direction of the bias is very difficult to determine, given the complex nature of the syndication decision. The model developed in Sørensen (2007) directly addresses this endogeneity problem by using a matching model to measure the effect that shifts in relative ranking, for agents on either side of the market, have on the quality of their match partners. Although the characteristics of other agents on the same side of the market changes the quality of a match partner, they do not affect the outcome of the match partner. Thus, the matching model provides the necessary exogenous variation. While this approach allows the econometrician to examine a wide variety of questions, it still does not deal with the endogeneity problems presented by syndication. The issue that remains unresolved is the aforementioned simultaneity bias.
One potential solution is to use the method in Sørensen (2007), and simply instrument for syndication. Of course, this strategy still suffers from the lack of available instruments. I propose a model that circumvents this simultaneity issue of syndication choice, while still accounting for the endogeneity caused by sorting.

The structural model consists of an outcome equation and a selection equation. Similar to Sørensen (2007), the outcome equation evaluates each company outcome, while the selection equation is a two-sided matching model. However, instead of the standard matching model developed in Gale and Shapley (1962) and Roth and Sotomayor (1990), I adapt the matching model with contracts from Hatfield and Milgrom (2005) to an empirical setting. In the model, the two types of contracts that can be offered are either syndication or non-syndication. The selection equation in the model deals with the simultaneity issue, while also controlling for the endogeneity due to sorting. Consequently, the bias in the outcome equation estimates is corrected. The method developed in the paper builds upon recent papers addressing endogeneity in outcome estimates using matching market selection equations (Sørensen, 2007; Chen, 2013; Akkus et al., 2021; Xia, 2019), and provides a framework for controlling for sorting when the matching market involves bilateral contracting between agents. Although these models are similar to Heckman (1979), the interaction between agents’ decisions in the matching market requires simultaneous evaluation of the integrals in the likelihood function. Since evaluating the likelihood function is numerically demanding, alternative popular matching estimators are rejected in favor of the use of Bayesian estimation techniques using Gibbs sampling (Gelfand and Smith, 1990; Geweke et al., 2003; Koop, 2003).

Although the method described above progresses the literature, it is still subject to some limitations. Foremost of these is the static nature of the matching model, when the VC industry is highly dynamic. There is a great deal of information between the initial investment in a company and its outcome that cannot be used as a dynamic empirical matching model has not yet been developed. Second, the model does not explicitly account for the preferences of lead VCs over non-lead VCs in the matching market. More specifically, the model assumes the market matches many companies to one VC, when in reality it has many companies matching to many VCs. Assuming a many-to-one matching model instead of a many-to-many matching model simplifies the analysis, and makes it possible to obtain point estimates. Another feature of matching models is that it assumes both sides have complete information, an issue that has only begun to be addressed recently in the theoretical literature. Finally, the numerically intensive nature of the model makes the estimation of extensive fixed effects intractable. However, the model is valid under these assumptions, and solves the issue of missing instruments and simultaneous causality bias when standard instrumentation cannot. The goal from an econometric standpoint is to offer a platform for future research on the power of matching models as tools to address complex endogeneity problems.

Before a preview of the findings in the paper, I provide a deeper discussion about the relationships between the forces in the model. The two most important forces are: (i) the true value-added from syndication, $VA$, which refers to the increase in company IPO probability due the benefits from syndication, and (ii) the selection effect due to sorting, $SE$, which represents the degree of bias in estimates of $VA$ that fail to address the endogeneity from sorting. More explicitly, Biased $VA = SE + VA$. The relationship between these three measures is depicted at the top of Figure 1, which links all the relevant forces in the

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3There are two largely popular techniques in the empirical matching literature. The class of estimators used in Fox (2010), Fox et al. (2015, WP), and Fox (2018) is especially useful in many-to-many matching markets, but assumes transferable utility (TU). The other estimator, developed in Agarwal (2015) and Agarwal and Diamond (2017), utilizes novel moment conditions to identify preferences for both sides of the market. The downside of both estimators is that the integrals that need to be evaluated cannot be combined with the outcome equation and computed directly.
model. Notice that I separate the SE into two parts. One is the use of syndicated contracts as a selection mechanism in the matching market, which I call the syndication selection effect (SSE). The effect coming from sorting along other characteristics is loosely referred to as ‘Other Sorting’. The SSE is of particular interest when examining the impact of syndicate formation. I further disaggregate the SSE into VC SSE and Company SSE. These encapsulate the effect of VC and company preferences over syndication on the SSE. I leave a more detailed explanation of the underlying mechanism of SSE, VC SSE, and Company SSE for Section 2. Since the three main motives for syndicate formation determine a VC’s

Figure 1: Effect of Syndication. Notes: (i) The arrows indicate direction of influence for one force on another. (ii) The Company VA and VC VA appear along arrows, because these arrows represent the influence of the VA on the Company and VC SSEs respectively.

preference for syndicating a particular investment, they each contribute to the VC SSE. However, notice that the VC VA is influenced by the VA, thus, it is important to expand on the difference between these two terms.\(^4\) The VC VA represents the additional return a VC expects to receive from syndicating an investment compared to investing alone. The VA refers to the value-added from syndication over non-syndication measured by the change in IPO probability based upon realized matches. In both cases, the value-added is attributed to benefits from syndication, such as, better management and advice. When VCs assess the value of an investment, they form expectations about the potential return on their investment. These expectations are based upon their beliefs about the VA, which leads to it influencing the VC VA. Additionally, following the path of influence for the VC VA, in Figure 1, one can see that VA is not even indirectly affected by VC VA. On the other hand, the Biased VA measure is indirectly influenced by VC VA, which is due to the biased measure failing to account for the underlying mechanism leading to a match. In the context of a Bayesian game, the VC VA is the \(\text{ex ante}\) valuation of an action based upon the VC’s beliefs, which are informed by the true \(\text{ex post}\) value, the VA.\(^5\) Although the \(\text{ex ante}\) value of an action is influenced by the value of the action \(\text{ex post}\) through belief formation, the reverse is not true. However, if one only observes the outcome of the game, and not the underlying beliefs, the value of the action will be measured incorrectly. The complexity of the issue is magnified when sorting also plays a role in determining outcomes, as seen in

\(^4\)Since the Company SSE is influenced by Company VA in a similar way to the VC counterparts, I focus on the case for VCs.

\(^5\)Here \(\text{ex ante}\) means prior to the initial investment, \(\text{ex post}\) refers to after the company outcome is realized, and the action is the decision to syndicate an investment.
Figure 1, which necessitates the estimation of a two-sided matching model. In the empirical analysis, I utilize the SDC VentureXpert database to compile a sample of 74 VCs investing in 2,665 companies from 2004 to 2012. I find that the value-added (VA) from syndication increases the probability of a company achieving and IPO by 2.07%. Additionally, I show that failing to account for the sorting between VCs and companies, causes estimates for the VA of syndication to be highly volatile across the sample period. This volatility results in an overestimation of the VA by nearly 40% for some years, and an underestimation of over 40% in others. Furthermore, I illustrate how the VC SSE, Company SSE, and the motives for syndication influence the size and direction of the bias. Finally, I determine the change in bias over time is largely due to sorting along unobservable characteristics.

The paper proceeds as follows. Section 2 provides a simple example of the two endogeneity issues along with a deeper discussion of the three motives for syndication. Section 3 introduces the two-sided matching model, Section 4 discusses the structural model, Section 5 covers the data and descriptive statistics, Section 6 presents the main findings, and Section 8 summarizes the paper.

2. Bias and Contracts

Outcome Bias. This section presents an example designed to illustrate the two sources of endogeneity plaguing estimation of the value-added from syndication. Consider a simple model in which the quality of companies and VCs are given by a single variable. In the market there are three VCs, \( j \in \{A, C, D\} \) with qualities \( Z_A = 5, Z_C = 2, \) and \( Z_D = 1 \). On the other side of the market there are three companies denoted \( C_i \) with quality \( X_i = 5 - i \), for \( i \in \{1, 2, 3\} \). Suppose the quality of VCs is completely determined by their experience, so that more experienced VCs are of higher quality. The quality of companies is assumed to be unobservable to the econometrician, but observable to VCs and other companies. As a result, there is sorting between VCs and companies along characteristics that the econometrician cannot observe. The assumption that only one side of the market has unobservable characteristics simplifies the example for the sake of intuition, but in practice each side of the market has observable and unobservable characteristics that are valued by the other side of the market. The outcome for a matched pair is determined by the following outcome equation:

\[
Y_{ij} = \gamma_0 + c_{ij} \gamma_c + X_i \gamma_x + Z_j \gamma_z + e_{ij}
\]  

(1)

where \( c_{ij} \) is an indicator that is equal to one if \((i, j)\) chose a syndicated contract and zero if they do not. Suppose that the true values of the coefficients are \( \gamma_0 = 0, \gamma_c = 15, \) and \( \gamma_x = \gamma_z = 10 \). It can be assumed that \( e_{ij} = 0 \) without loss of generality. An equilibrium match is defined by a VC-company-contract triple, \((i, j, c)\), \( c \in \{s, n\} \), where \( s \) and \( n \) represent syndicated and non-syndicated contracts respectively. Let the equilibrium matching in the market be \(\{(1, A, n), (2, C, s), (3, D, n)\}\) with outcome values 90, 65, and 30. The outcome value and corresponding contract for a given matched pair are contained in Table 1. The outcome values are implied by the true values of the coefficients and are observable to the econometrician. Thus, when both VC and company quality are unobserved, the outcome

\[\text{It should be noted that the assumption of complete information in the matching market does not change the fact that VCs form expectations over the value of an investment, but rather that these expectations are the same and equal to the \textit{ex post} value of the investment. Under this assumption, the beliefs do not bias measurement of the outcome directly, and as such, there is no arrow originating at the VC VA going directly to the Biased VA. However, they do impact the measurement of the outcome indirectly through their influence in the matching market.}\]
equation estimated by the econometrician is:

\[ Y_{ij} = \hat{\gamma}_0 + c_{ij}\hat{\gamma}_c + Z_j\hat{\gamma}_z + e_{ij} \]  

(2)

where the OLS estimates from (2) are \( \hat{\gamma}_0 = 10 \), \( \hat{\gamma}_c = 20 \), and \( \hat{\gamma}_z = 15 \). In this case, the influence of VC experience is biased upward by 5, and the value-added from syndication is biased upward by 5. The bias originates from VCs with better characteristics matching with companies that have better unobservable characteristics. Although we know that there are no unobservable VC characteristics, it is not possible to know this in practice. The presence of unobservables on both sides of the market requires additional variation in order to identify the coefficients. This issue can be addressed by observing an additional market. Suppose a second market is observed with an additional VC denoted, \( VC_B \), with \( Z_B = 4 \), and a new company \( C_4 \). Let the equilibrium matching be \( \{(1, A, s), (2, B, n), (3, C, s), (4, D, n)\} \). The entry of \( VC_B \) pushes both \( VC_C \) and \( VC_D \) down in the preference rankings of companies. As a result, companies \( C_2 \) and \( C_3 \) have different match partners, which provides the necessary exogenous variation. In order to utilize the additional variation, a matching model is required to determine preference rankings for each side of the market.

**Table 1: Bias Example**

<table>
<thead>
<tr>
<th>Market 1</th>
<th></th>
<th>Market 2</th>
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<tbody>
<tr>
<td></td>
<td>( VC_A )</td>
<td>( VC_B )</td>
<td>( VC_C )</td>
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<tr>
<td>( C_1 )</td>
<td>(90,n)</td>
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<td>-</td>
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<tr>
<td>( C_2 )</td>
<td>-</td>
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<td>(65,s)</td>
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<tr>
<td>( C_3 )</td>
<td>-</td>
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<tr>
<td>( C_4 )</td>
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</table>

Notes: Each cell contains the outcome value and contract of a match. For example, a match between \( VC_A \) and \( C_1 \) results in an outcome value of 90 under a non-syndicated contract. A dash indicates that a match between the VC and company did not occur.

**Contracts.** Thus far, I have not addressed the importance of syndication as a sorting mechanism, and the effect it has on outcome estimates. Since the decision to syndicate an investment is determined when a VC and company match, an endogeneity issue arises in the form of a simultaneity bias. The nature of the VC industry makes it nearly impossible to find acceptable instruments, so another solution is required in order to deal with the endogeneity problem. Treating the decision to syndicate as a choice between contracts addresses the endogeneity issue directly and has precedence in the literature, (Cassamatta and Haritchabalet, 2007; Hellmann, 2002). Consider again the two markets used in the example of the outcome bias. The variation across the two markets is easily explained by a matching model that treats syndication as a contract choice, but cannot be explained by a model that does not. Aside from \( VC_C \) and \( VC_D \) being pushed down in the preference rankings, the other notable difference across the two markets is that the entry of \( VC_B \) leads to \( VC_A \) switching from a non-syndicated contract to a syndicated contract. The implication in a model with contracts is that competition from \( VC_B \) forces \( VC_A \) to offer a syndicated contract. This provides information about preferences over contracts for not only VCs, but also for companies. On the other hand, a standard matching model cannot determine VC preferences using the change in equilibrium contracts for \( VC_A \). For example, suppose the utilities derived from a match for VCs and companies, respectively, are given by:

\[ v_{ij}^c = X_i\alpha_x + c_{ij}\alpha_c + \varepsilon_i \] and \[ u_{ji}^c = Z_j\beta_z + c_{ij}\beta_c + \xi_j \]  

(3)
where again the error terms are assumed to be equal to zero (i.e. \( \varepsilon_i = \xi_j = 0 \)). Given these functional forms, the preferences inferred from the variation across the two matching markets under a model with contracts can pin down values for \( \alpha^c \) and \( \beta^c \) that do not include zero: \( \alpha^c \in (\alpha(X_2 - X_1), 0) \) and \( \beta^c \in \beta(Z_C - Z_D, Z_A - Z_C) \). On the other hand, a model without contracts can only give \( \beta^c \in \beta(Z_B - Z_A, Z_A - Z_C) \), which includes zero, and provides no information about \( \alpha^c \). Thus, the standard matching model cannot identify parameters in the simplest possible functional form for preferences. Of course, this implies that the estimates in the outcome equation of the model cannot be identified.

Now I discuss the relationship between the three motives for syndication and the selection and value-added effects in the structural model. The value-added effect is the true value of \( \gamma_c \) from (1). This is the impact of syndication on the realized value of the company. In my model, it is measured as an increase in the probability of an IPO or acquisition, while it could also be measured as an increase in rate of return as in Brander et al. (2002). The selection effect is the difference between the true value of syndication and the estimate from a model that does not account for sorting. However, this selection effect is not the selection effect resulting from the availability of syndicated and non-syndicated contracts. It is comprised of the syndication selection effect (SSE), the pure selection effect (PSE), the finance-related motive, and additional sorting along other characteristics. In order to understand how these are all interrelated, a discussion of the underlying theory on matching with contracts is required; see Serfes (2008); Alonso-Paulí and Pérez-Castrillo (2012); Macho-Stadler et al. (2014) for some examples. Typically, the literature assumes that agents first match and then choose the contract that they prefer. Consequently, these models can be solved through backward induction, which derives the utilities for each agent under each contract with all potential match partners. The utilities in (3), are the empirical analog of the solution to the contracting problem. In Cassamatta and Haritchabalet (2007), the utility is necessarily a function of the screening ability of the VCs, and thus related to the PSE. The same logic can be used to see how the finance-related motive will also impact the empirical utilities. The SSE refers to the use of syndicated contracts to attract match partners, which occurred when \( VC_B \) entered Market 2 in the example. Thus, I consider the SSE to be a measure of the use of syndication as a selection mechanism. I discuss the computation of the VA and the various components of the SE when I introduce the structural model in Section 4. From this section it is evident that the effect contract competition has on the outcome estimates is not easy to predict, especially if unobservables on each side of the market play a role in contract choice. Additionally, the failure to account for the impact of contract choice on the matching market equilibrium will lead to incorrect estimates of the influence of syndication on sorting. Therefore, disentangling the SE from the VA requires the estimation of a model that controls for both the sorting along unobservables as well as the use of syndication as a selection mechanism.

3. Two-Sided Matching Model

Before introducing the structural model, I discuss the underlying theory guiding the estimation of the model. The model builds upon Sørensen (2007) and Chen (2013) by extending the setting to a matching model with contracts developed in Hatfield and Milgrom (2005).

3.1. Model Assumptions

The main assumption is that the investment decisions in the VC market can be modeled as a static one-to-many matching model with contracts. Each VC can match with multiple...
companies, but a company is only capable of matching with a single VC. It is fairly common for multiple VCs invest in a company within a given round, but also over multiple rounds. Sørensen (2007) chooses to address this issue by assuming the matching occurs in the initial round between the company and the lead investor, since the lead VC is the most involved in everyday management of the company. The restriction to the initial investment round is due to both econometric limitations and industry specifics. The impact of investors in later rounds on company success is not negligible and the lead investor can even change over the life of a company, (Cumming and Dai, 2013), but the econometric method required to estimate a dynamic matching model has not yet been developed. However, the severity of this issue is alleviated somewhat by the fact that the lead VC in the first round influences the VCs that make investments in later rounds. The decision to model the syndication of an investment as choice between contracts rests on the assumption that information about the lead investor is sufficient to determine VC preferences across contracts. As mentioned before, Lerner (1994) finds that early round investors syndicate with similarly experienced VCs, which provides support for a model with contracts.

Additionally, I assume that VCs have a limit on the number of companies in which they can invest in a given year. This could be viewed as either a time or capital constraints. The limit for each VC is assumed to be equal to the number of observed matches and is especially important in the VC market, which does not have data on unmatched companies. Furthermore, the VCs and companies are assumed to have complete information on the preference rankings of each side of the market, which is a standard assumption in two-sided matching models. This does not imply that all VCs know the true valuation of a company, but rather that VCs know how all the other VCs rank a given company. Finally, the last major assumption is that the model is one of non-transferable utility (NTU), which rules out the use of transfers. This assumption prevents a low quality VC from attracting a company by offering a higher share than a high quality VC. In a model with contracts, this implies that the sharing rule within each contract is fixed. In other words, the sharing rule for solo and syndicated contracts are fixed, but they need not be the same. While this may seem restrictive, Sørensen (2007) notes that it is a feature of the VC industry, since renegotiation in later rounds makes it difficult to commit to transfers in the first round of investment.8 Thus, an NTU model is viewed as more appropriate for the VC industry.

3.2. Agents and Matchings

A given market, \( t = 1, 2, \ldots, T \), is modeled as two disjoint sets of agents, with a company denoted by \( i \in I_t \) and a VC by \( j \in J_t \); and a set of contracts, where a contract is denoted by \( c \in C \). The set of potential matches in a given market is \( M_t = I_t \times J_t \times C \). I proceed by focusing on a single market, and remove the market subscripts to simplify notation. A matching is denoted by \( \mu \), is the set of matches such that \( (i, j, c) \in \mu \) if and only if company \( i \) matches with \( j \) in the contract \( c \). Additionally, the VC matched to company \( i \) is given by \( \mu(i) \) and a company matched to VC \( j \) is given by \( \mu(j) \). Since the matching is one-to-many, \( \mu(i) \) is a singleton and \( \mu(j) \) is a set. Thus, we can say the VC matched to company \( i \) is \( j = \mu(i) \), while the same company is represented as \( i \in \mu(j) \). In the market, companies have preferences over VCs and VCs have preferences over companies. These preferences are mapped to utilities for each side of the market. The utility company \( i \) gets from matching with VC \( j \) in contract \( c \) is given by \( u_{ji}^{c} \). The utility VC, \( j \), obtains from investing in company \( i \) in contract \( c \) is given by \( v_{ij}^{c} \). The mapping of preferences to utilities makes it possible to compare the value of a given match, for each side of the market, to an alternative potential match. For example, company \( i \) prefers matching with \( j \) in contract \( c \) to some alternative VC, \( j' \) in the same contract if \( u_{ji}^{c} > u_{ji'}^{c} \). Similarly, the VC prefers a match with \( (i, c) \) to the alternative

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8This assumption rules out the use of the class of estimators developed in Fox (2010), Fox et al. (2015, WP), and Fox (2018) as they assume TU.
\((i', c)\) if \(v_{ij}^{*} > v_{i'j}^{*}\). This modeling approach follows Chen (2013), and is necessary in order to estimate a matching model with contracts. The implication of this setup is that the model is NTU, as mentioned previously, because there is no sharing of a joint surplus through the transfer of utilities from one side of the market to the other.

3.3. Equilibrium/Solution Concept

The standard equilibrium concept used in the matching literature is stability. A matching is stable if no agent prefers to deviate from the matching to form a new match, but there are multiple types of stability that dictate the types of deviations that can be considered. In a many-to-one matching model, it is possible to conceive of deviations involving multiple agents (i.e. multiple companies and a single VC). A matching that is robust to this type of deviation is referred to as group or set-wise stable, and presents complications in determining the stability of a given matching. However, in the college admissions model, Roth and Sotomayor (1990) show that the much simpler condition of pair-wise stability implies group stability. Hatfield and Milgrom (2005) extend the concept of pair-wise stability to a setting with contracts, and provide results that ensure existence and uniqueness of a pair-wise stable equilibrium. The issue of uniqueness is of particular interest in an empirical setting, as it allows for point estimation of the structural model. The definition of pair-wise stability in the context of the model is given below. First, let the equilibrium vector of syndication contracts offered by a given VC \(j\) is \(c_j^{*} \subseteq C_j\), where \(C_j\) is the total set of contracts VC \(j\) can offer.

### Definition 1 (Pair-wise Stability)

A matching \(\mu\) is pair-wise stable if it satisfies individual rationality and the no blocking condition:

1. **Individual Rationality:**
   - (i) **Companies:** \(\mu(i) \succeq_i \{\emptyset\} \quad \forall i\)
   - (ii) **VCs:**
     - (a) \(\mu(j) \succeq_j \mu(j) \setminus \{i\} \quad \forall i \in \mu(j)\)
     - (b) \(\exists c'_j \subseteq C_j \text{ s.t. } v(\mu' c'_j) \geq v(\mu c_j^{*})\)
     - (c) \(|\mu(j)| \leq q_j\)

2. **No Blocking:** If \(j \succ_i \mu(i)\)
   - (i) Then \(\mu(j) \succeq_j \{\mu(j) \setminus \{i'\}\} \cup \{i\} \quad \forall i' \in \mu(j)\)
   - (ii) And if \(|\mu(j)| < q_j\), then \(\mu(j) \succeq_j \mu(j) \cup \{i\} \quad \forall i \notin \mu(j)\)

The individual rationality condition for companies simply requires that companies prefer their match to remaining unmatched. The individual rationality constraints in (a) and (c) are the same as in the classic many-to-one matching model. They require VCs to prefer the set of companies with whom they are matched in equilibrium to removing any company from their portfolio and prevent them from exceeding their quota. The condition in (b) is unique to a model with contracts. It states that there is not another set of contracts that a VC can choose that will result in them obtaining higher utility. The no blocking conditions are fairly straightforward. If a company \(i\) prefers a VC \(j\) to its equilibrium match partner, then it must be that the VC does not wish to replace any company in its portfolio with company \(i\) when the VC has fulfilled its quota. If the VC has not fulfilled its quota, then it must be that the VC does not wish to add company \(i\) to its portfolio.

---

9The total set of contracts a given VC \(j\) can offer is \(C_j \equiv \bigcup_{i \in I} \{c_i\} \cup \{\emptyset\}\).
As with Sørensen (2007) and Chen (2013), the unique equilibrium matching can be characterized by a set of inequalities derived using the definition of pair-wise stability. However, a further assumption is required to fully characterize the equilibrium matching using these inequalities in the presence of contracts. More specifically, the side of the market making contract offers impacts the set of inequalities. In a model without contracts, the set of inequalities can be derived in the same manner irrespective of whether the equilibrium is assumed to arise from the VC- or company-offering Gale-Shapley algorithm. Although the generalized Gale-Shapley algorithm from Hatfield and Milgrom (2005) coincides with the Gale-Shapley algorithm when only one type of contract is available for any potential match, they differ if even one potential match has multiple contract options. In order to see how this is the case, consider the following example:

**Example 1** Consider a one-to-one matching market with a set of VCs, $J = \{j_1, j_2, \ldots, j_J\}$, and a set of companies, $I = \{i_1, i_2, \ldots, i_I\}$, in which the set of contracts for each potential match is a singleton, $c$, except for a match between VC $j_1$ and company $i_1$ who can choose between $c$ and $c'$. Suppose all agents on each side of the market have identical preferences, except that VCs prefer $c$ and companies prefer $c'$. Consider the preferences for $j_1$ and $i_1$, denoted by $P(j_1)$ and $P(i_1)$ respectively:

- $P(j_1): (i_1, c) \succeq (i_1, c') \succeq (i_2, c) \succeq \cdots \succeq (i_J, c)$
- $P(i_1): (j_1, c') \succeq (j_1, c) \succeq (j_2, c) \succeq \cdots \succeq (j_J, c)$

Given these preferences, a VC-offering algorithm would result in $\mu = (i_1, j_1, c) \in \mu$, and a company offering-algorithm would lead to $\mu' = (i_1, j_1, c') \in \mu$. If the contract choice for this VC-company pair were a singleton, the equilibrium matching would be the same regardless of the side making the offers. The mechanism driving this change in the observed equilibrium matching provides information about preferences over contracts for both sides of the market, which ultimately enables identification of parameters modeling these preferences.

In estimation of the structural model, I assume that VCs make contract offers to companies. This is a natural assumption in the VC market as VC firms identify companies, set up meetings, and eventually offer funding for a share of the company. The set of inequalities that characterize an equilibrium matching is used to derive lower and upper bounds on the utilities for VCs and companies. The derivation of these bounds can be found in Appendix A. These bounds are denoted by $\underline{u}_{ji}$, $\overline{u}_{ji}$, $\underline{v}_{ij}$, and $\overline{v}_{ij}$.

**Proposition 1** The equilibrium matching, $\mu$, is stable if:

1. $U \in \mathbb{U}_\mu \iff \underline{u}_{ji} \in (\underline{u}_{ji}, \overline{u}_{ji}) \quad \forall (i, j, c) \in M$
2. $V \in \mathbb{V}_\mu \iff \underline{v}_{ij} \in (\underline{v}_{ij}, \overline{v}_{ij}) \quad \forall (i, j, c) \in M$

where $U$ and $V$ are vectors containing all the utilities in the market for companies and VCs respectively, and $\mathbb{U}_\mu$ and $\mathbb{V}_\mu$ are the sets of utilities for which $\mu$ is the unique stable matching.

---

10 Of course the VC- and company-offering algorithms still converge to the VC and company preferred allocations in the core respectively.

11 The contract available for each potential match need not be the same. What is important is that the set of potential contracts for a VC-company pair is a singleton.
4. Empirical Structural Model

The structural model is composed of two parts. The first portion is the matching model. The preferences for each set of agents are mapped to latent utilities. The latent utilities for a potential match \((i, j, c) \in M\) are given by:

\[
\begin{align*}
    u_{ji}^c &= Z_{ji}^c \beta + \xi_j^c \\
v_{ij}^c &= X_{ij}^c \alpha + \varepsilon_{ij}^c
\end{align*}
\]

where the utility the company obtains from the match is composed of a vector of observable company-VC-contract characteristics valued by companies, \(Z_{ji}^c\); as well as unobservable VC-contract characteristics, denoted \(\xi_j^c\). The utility obtained by the VC is comprised of a vector of observable company-VC-contract characteristics valued by VCs, \(X_{ij}^c\); in addition to the match-specific unobservables given by \(\varepsilon_{ij}^c\). The functional forms for the utilities can be substituted into the two conditions in Proposition 1 to obtain the set of error terms for which the matching, \(\mu\), will be stable. These two sets of error terms are given by:

\[
\begin{align*}
    \xi &\in \Xi \equiv U_\mu - Z \beta \\
    \varepsilon &\in \mathcal{E} \equiv V_\mu - X \alpha
\end{align*}
\]

where \(\xi\), \(\varepsilon\), \(Z\beta\), and \(X\alpha\) are all \(|M| \times 1\) vectors. The likelihood function of the matching model for a given market is given by:

\[
L(\mu; \alpha, \beta) = Pr(\xi \in \Xi, \varepsilon \in \mathcal{E}) = \int \int 1 \{\xi \in \Xi, \varepsilon \in \mathcal{E}\} dF(\xi, \varepsilon)
\]

where the likelihood function for multiple markets is just the product over all the markets. The model is a discrete choice model, and it is only identified up to location and scale. The implication is that variables which are constant within a market cannot be identified, so they are not included in the utility specifications. The scale is normalized by set the variance of the error terms in each latent utility equal to one.

**Outcome Equation.** The second portion of the structural model is the outcome equation. The outcome for a given matched triple \((i, j, c) \in \mu\) is given by:

\[
Y_{ij} = W_{ij}^c \Gamma + e_{ij}
\]

where \(W_{ij}^c\) contains \(X_{ij}^c\), \(Z_{ji}^c\), and additional observed characteristics not included in the matching model. The vector of parameters is denoted by \(\Gamma\), and the unobservables that affect the outcome of a matched pair are given by \(e_{ij}\). In the case of the binary outcome, \(Y_{ij}^*\) is a latent variable:

\[
Y_{ij} = \begin{cases} 
1 & \text{if } Y_{ij}^* \geq 0 \\
0 & \text{Otherwise}
\end{cases}
\]

The distinction between the observed binary variable and the latent variable is the same as in a standard Probit, but is more important in a Bayesian model for reasons that will be covered in Section 4.2. It is still the case that the sign and location of the parameters are identified, while it is necessary to make a scale normalization. This is commonly done by setting the variance of the error term equal to one, but the approach in the current model is slightly different. I discuss the modeling of the error terms in the subsequent section.
4.1. Distribution of Error Terms

The model makes the standard assumption that the error terms in a match, \( \xi_j \), \( \varepsilon_{ij} \), and \( e_{ij} \) are not correlated with the observable characteristics: \( W_{ij} \), \( X_{ij} \), and \( Z_{ji} \). As mentioned previously, the variances for the error terms in the latent utilities are set equal to one. The error term from the outcome equation is given by:

\[
e_{ij} = \delta \varepsilon_{ij}^c + \kappa \xi_j + \nu_{ij}, \quad \nu_{ij} \sim N(0,1)
\]

which allows for correlation between the error terms in the matching model and outcome equation. The scale normalization in the outcome equation is achieved by setting the variance of \( \nu_{ij} \) equal to one. The error terms are assumed to follow a multivariate normal distribution, while being independent across matches, that is summarized below:

\[
\begin{pmatrix}
e_{ij} \\
\xi_j \\
\varepsilon_{ij}
\end{pmatrix}
\sim N\left(0, \begin{bmatrix}
\delta^2 + \kappa^2 + 1 & \delta & \kappa \\
\delta & 1 & 0 \\
\kappa & 0 & 1
\end{bmatrix}\right)
\]

The covariance between the error terms in the latent utilities and the error term in the outcome equation captures unobservable characteristics that affect both the outcome equation and the latent utilities. These characteristics are not observable to the econometrician, but are observable to the VC and company. For example, a company may have financial information that is only observable to the VC, which makes the company more desirable and more likely to achieve a successful outcome. As a result, the correlation between the observables will be strong and positive. Alternatively, a VC may have a prior relationship with the CEO of the company that induces an investment, but has no effect on the outcome of the company. In this case, the correlation between the error terms will be positive but negligible. Thus, the coefficients \( \delta \) and \( \kappa \) capture the degree of sorting on unobservables.

4.2. Estimation and Identification

In a standard Probit model, the likelihood function can be factored into a product set of likelihood functions for each observation. The same would be possible in the current setting if the matching model were simply a binary outcome indicating whether or not a VC and company were an observed match. However, in the structural matching model, the investment decisions by each VC impact the decisions made by the other VCs in the market. The implication is that the individual likelihood functions are no longer independent, and the integrals must be evaluated simultaneously. The high dimensionality of these integrals makes it impossible to directly estimate the likelihood function using maximum likelihood. The solution to this problem is to use Bayesian estimation. Bayesian inference of the model is obtained using data augmentation and a Gibbs sampling algorithm to perform Markov Chain Monte Carlo (MCMC) simulations. As noted in Sørensen (2007), the difficulties posed by incorporating a matching model provide a solution to the endogeneity issue. The influence on a VC’s investment decision from the characteristics of other agents in the market provides exogenous variation that makes it possible for unbiased identification of the model parameters. This is particularly important in the VC industry, because it is notoriously difficult to find acceptable instruments. The decision to model the choice of syndication as a matching model with contracts is motivated by the lack of instruments and the exogenous variation provided by a matching model. In the case of syndication, a VC’s decision to syndicate is affected by the choices of other VCs in the market, because they
influence the type of contract a VC must offer a company in order for the company to choose them over other VCs. Of course, this works in the opposite direction as well. A company may find accepting their less preferred contract with a given VC to be a better option than any contract with a different VC. Recall the example in the introduction, in which the impact of competition between VCs using syndication contracts to entice companies influences the set of equilibrium contracts. Not only do instrumental variables fail to solve this problem, introducing an intermediate equation of syndication choice does not remedy the issue either. The sole method of alleviating the problem is to model the matching decision and contract choice simultaneously. Thus, a matching model with contracts is necessary in order to obtain unbiased estimates.

Prior Distributions. It is necessary to impose prior distributions on the parameters in Bayesian estimation. All the parameters are assumed to follow independent normal distributions. These distributions are denoted \( N(\pi, \Sigma_\pi) \), \( N(\beta, \Sigma_\beta) \), \( N(\Gamma, \Sigma_\Gamma) \), \( N(\delta, \Sigma_\delta) \), and \( N(\kappa, \Sigma_\kappa) \). The implication of choosing a normal distribution for the priors of the parameters and the error terms is that the posterior distributions are also normal, and greatly simplifies the estimation of the model. Additionally, in order to identify the sign of the parameters in the model, a sign restriction must be placed on one of them. I choose to restrict \( \delta \) to be negative, so that the prior distribution for \( \delta \) is normal distribution that is truncated from above at zero. I follow \textcite{Sorensen2007} and \textcite{Chen2013} in choosing uninformative priors. More specifically, the prior means and covariances are assumed to be zero, and the variances are all 10. The prior variance is no less than 60 times larger than the posterior variances from above at zero. I follow Sørensen (2007) and Chen (2013) in choosing uninformative priors. More specifically, the prior means and covariances are assumed to be zero, and the variances are all 10. The prior variance is no less than 60 times larger than the posterior variance of each parameter, with it being over 1,000 times larger for most parameters. This implies that the information from the Bayesian inference is substantial and largely based upon the data.

Conditional Posterior Distributions. I have already discussed the likelihood function for the matching model, but that is only a portion of the likelihood function used in estimation. Now I will introduce the joint density function used to construct the likelihood function for the matching model, but that is only a portion of the likelihood function used in estimation. The reason that standard maximum likelihood is not feasible can be seen in the indicator function in (14). Maximum likelihood would require factoring both \( U_\mu \) and \( V_\mu \) into a product integrals for each VC and company.\(^{12}\) However, since investment decisions by agents are interrelated within a market, whether or not a given utility for each agent is in the set of stable utilities depends upon all the other agents in the market.

The conditional posterior distribution of the latent variables and the parameters can be obtained using Bayes’ rule:

\[
p(Y^*, U, V | \Omega, \theta) = p(\theta) \times p(Y^*, U, V | \Omega, \theta) / p(\mu | \Omega) \\
\propto p(\theta) \times p(Y^*, U, V | \Omega, \theta)
\]  

(15)

where \( p(\theta) \) is the prior densities of the parameters. The conditional posterior distributions are derived in Appendix B.

\(^{12}\)Recall, \( \Xi \equiv U_\mu - Z_\beta \) and \( \varepsilon \equiv V_\mu - X_\alpha \).
**Simulation.** The Gibbs sampling algorithm evaluates parameters in blocks. Each of the parameters constitutes a block (i.e. $\alpha$, $\beta$, $\Gamma$, $\delta$, and $\kappa$), and each of the latent variables is a block. Thus, the number of blocks is $\sum_{t=1}^{T} (|I_t| + C (|I_t| + |J_t|)) + 5$. The algorithm samples each block conditional on the other blocks in each iteration, and eventually converges to the joint distribution. The Gibbs sampler takes 20,000 initial draws 2,000 of which are discarded for burn-in. The estimation results presented in Section 6 use the draws remaining after further discarding and thinning based upon the statistical tests considered below.

I discuss two tests to confirm the accuracy and convergence of the MCMC algorithm. The first is the Raftery-Lewis test (Raftery and Lewis, 1992), which determines if the desired accuracy is possible using all the draws. I follow convention by examining the 95% highest posterior density intervals to ensure that they have probabilities between .94 and .96 with probability .95. When using all the draws to conduct the test, I find that the burn-in is relatively small (23) and the total number of draws necessary is 5,759, while only every third draw should be kept. Before I implement the required thinning, I drop the first 10% of the draws to allow for burn-in. I test the accuracy again after thinning to determine if the draws are acceptable, and find that the burn-in is 7, the total number of necessary draws is 1,866, and no additional thinning is required. The second test I conduct uses results from (Geweke, 1992) in order to verify that the MCMC algorithm has converged. I discard the first 100 draws from the thinned sample as additional burn-in, and compute Geweke’s convergence diagnostic for each parameter based on the first 10% of draws after burn-in and the last 50% after burn-in. The results show that the convergence diagnostic implies the parameter means from the two sub-samples are not different at the 95% level, which indicates that the MCMC algorithm has converged.

5. Data and Descriptive Statistics

In this section, I will discuss the sample used in estimation and the construction of particular variables of interest. The dataset used in the estimation of the model is compiled from the SDC Platinum VentureXpert database. The database contains investments from 1962 up to present day, and has been used in previous studies (Sørensen, 2007; Gompers et al., 2008, 2009). Although data is available all the way back to 1962, I use a subset starting with investments made in 2004 up to investments made in 2012. However, I use investments starting in 1975 to create variables that are updated yearly. I follow Sørensen (2007) in limiting the sample so companies are given at least seven years to go public. Additionally, I follow the procedure of the author in restricting the sample to the first investment made by the lead investor in the company. The lead investor is defined as the first investor in the company making the largest total investment. When multiple VCs participate in the initial investment round, which is the case for syndicated investments, the VC firm making the largest initial investment is considered to be the lead investor. I then restrict observations to include only investments in companies from the top quartile of states with the most investments over the course of the sample. I define a matching market as a single year. Thus, all the VC firms in the sample making investments in the selected states over the course of a year are considered to be competing with each other in a matching market. I consider all of these states a single market, since there is large amount of interstate investment.

Of all investments in the sample, 69% are made within the same state. Additionally, the median distance between company and VC zip codes for out of state investments is 2,074 miles. Therefore, it seems sensible to assume that VC firms, especially the largest

---

13The first 10% and last 50% samples are the conventional proportions when comparing parameter means.
and most active, compete for companies on a nationwide scale.

I keep only the VC firms that make more than 20 investments in the remaining states from 2004-2012, and remove VCs that are not consistently active over the course of the sample. I do this to ensure that the VCs in the sample are the largest and most active investors in the VC market. I also only keep VC firms that are designated as private equity firms, which removes firms that are affiliated with government investment funds or banks. I keep VCs that invested in at least two companies in a given market, and matching markets in which there are at least two VCs. All of these restrictions result in a sample with 2,665 companies and 74 VCs.

Table 2: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Company-Level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPO</td>
<td>2,665</td>
<td>.065</td>
<td>.246</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Syndication</td>
<td>2,665</td>
<td>.544</td>
<td>.498</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PE VC</td>
<td>2,665</td>
<td>1.946</td>
<td>1.182</td>
<td>1</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>Stage</td>
<td>2,665</td>
<td>.152</td>
<td>.359</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>IT</td>
<td>2,665</td>
<td>.786</td>
<td>.41</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Medical</td>
<td>2,665</td>
<td>.12</td>
<td>.325</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Non-High Technology</td>
<td>2,665</td>
<td>.094</td>
<td>.292</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Business-Facing</td>
<td>2,665</td>
<td>.391</td>
<td>.488</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>ln(Initial Investment)</td>
<td>2,665</td>
<td>8.007</td>
<td>1.541</td>
<td>.087</td>
<td>12.293</td>
<td>8.316</td>
</tr>
<tr>
<td>ln(Distance)</td>
<td>2,665</td>
<td>3.724</td>
<td>2.616</td>
<td>0</td>
<td>7.759</td>
<td>3.15</td>
</tr>
<tr>
<td><strong>Panel B: VC-Level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experience</td>
<td>486</td>
<td>75.628</td>
<td>62.374</td>
<td>0</td>
<td>345</td>
<td>61</td>
</tr>
<tr>
<td>IPO Rate</td>
<td>486</td>
<td>.111</td>
<td>.109</td>
<td>0</td>
<td>1</td>
<td>.097</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>486</td>
<td>.643</td>
<td>.232</td>
<td>0</td>
<td>1</td>
<td>.667</td>
</tr>
<tr>
<td>HHI_{jt}</td>
<td>486</td>
<td>.236</td>
<td>.136</td>
<td>0</td>
<td>1</td>
<td>.198</td>
</tr>
<tr>
<td>Net. Adjusted Capital</td>
<td>486</td>
<td>5.562</td>
<td>1.043</td>
<td>2.413</td>
<td>9.897</td>
<td>5.354</td>
</tr>
<tr>
<td>Portfolio Size</td>
<td>486</td>
<td>5.484</td>
<td>3.728</td>
<td>2</td>
<td>21</td>
<td>4</td>
</tr>
</tbody>
</table>

Notes: The sample contains investments in 2,665 companies over the course of the sample. In the sample, there are 486 distinct VC-year observations. IPO is equal to one if the portfolio company achieves goes public before 2020 and zero otherwise. Syndication is equal to one if an investment involves other VCs and zero otherwise. PE VC counts the number of VCs in the first investment round. Stage is equal to one if a portfolio company is classified as late stage at the time of investment and zero otherwise. IT, Medical, and Non-High Technology are equal to one if the portfolio company is in that industry and zero otherwise. Business-Facing is equal to one if the portfolio company’s main customers are other businesses and zero otherwise. ln(Initial Investment) is the log of the total dollar amount the portfolio company receives in its first round. ln(Distance) is the log of 1 plus the distance, in miles, between the zip code of the lead VC and the zip code of the portfolio company. Portfolio Size is the number of lead investments that a VC makes in a given year. The other VC characteristics are computed yearly on a rolling basis. I discuss each of these in detail in Section 5.1.

Although they are used as one of the main measures of success in the VC literature, one can see from Table 2 that IPOs are uncommon outcomes in the industry, with only 6.5% of the sample companies achieving an IPO. While the success rate is very low on average, the payoff of a single IPO is large for a VC. Thus, even though many of their investments may fail, top VC firms compensate for these failures by guiding a small number of companies to IPOs. The focus on syndication is also common throughout the VC literature, and the data justify the attention given to the subject; with 54.0% of the initial investments in the sample being syndicated. A further justification is that 69.9% of the IPOs in the sample
are achieved by companies receiving syndicated investments in the initial round compared to 53.3% of non-IPO outcomes involving a syndicated first round of investment.

Since the sample is focused on the initial investment in a company, it is not surprising that only 15.2% of the companies are late stage. Additionally, Table 2 shows that the VC industry is largely geared toward companies in the Information Technology industry during the sample. Investment in companies in the Medical and Non-High Tech industries is roughly the same throughout the sample.

In Panel B of Table 2, the observations are collapsed to a single VC-year, so that VCs making more investments in a year are not over represented. Experience variables are measured by counting the number of investments a VC has made prior to each year. The detailed explanations of the variables and how they are computed will be left for Section 5.1

Table 3: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Non-Syndicated</th>
<th>Syndicated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs</td>
<td>1,216</td>
<td>1,449</td>
</tr>
<tr>
<td>PE VC</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>IPO</td>
<td>.043</td>
<td>.084</td>
</tr>
<tr>
<td>Stage</td>
<td>.164</td>
<td>.142</td>
</tr>
<tr>
<td>Business-Facing</td>
<td>.376</td>
<td>.403</td>
</tr>
<tr>
<td>ln(Initial Investment)</td>
<td>7.431</td>
<td>8.49</td>
</tr>
<tr>
<td>Experience</td>
<td>82.725</td>
<td>87.937</td>
</tr>
<tr>
<td>IPO Rate</td>
<td>.073</td>
<td>.116</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>.663</td>
<td>.644</td>
</tr>
<tr>
<td>$HHI_{jt(5)}$</td>
<td>7.834</td>
<td>8.646</td>
</tr>
<tr>
<td>Net. Adjusted Capital</td>
<td>5.768</td>
<td>5.677</td>
</tr>
</tbody>
</table>

Notes: The table separates the investments into the sample of 1,216 for which Syndication is equal to zero, and the sample of 1,449 for which Syndication is equal to one. Here the statistics for the variables that are VC-year specific are computed at the portfolio company level (i.e. The experience value for a VC investing in 20 companies in a given year is counted 20 times.).

I report summary statistics for syndicated and non-syndicated investments for select variables in Table 3. The purpose of this table is to provide the reader with a clear view of the differences between solo and syndicated investments. Since an investment is defined as syndicated if more than one private equity firm invests in the first round, PE VC in

14The variables with minimum values of zero are all computed on a rolling basis. Thus, when a VC enters the sample for the first time, its value for the previous year is zero.
Panel A of Table 3 is equal to one for all observations. In Panel B, we see that the median syndicate size is equal to two, which implies that lead VCs typically bring in one additional VC when they syndicate. Finally, the disparity in IPO across contracts is reinforced by the fact that the probability of a syndicated investment resulting in an IPO is nearly twice that of a non-syndicated investment (4.3% vs. 8.4%).

5.1. Variables and Characteristics

In this section, I introduce the company and VC characteristics used in the matching model. I detail the construction of measures used in previous papers as well as those that are novel to the current paper. I finish by discussing syndication, and the syndicated contract heterogeneity terms.

Company Characteristics. The company-level data in the SDC database is more limited than data for VCs. I define three company industry indicators for the following industries: Information Technology (IT), Medical/Health/Life Science (Medical), and Non-High Technology (Non-High Tech). I also define a stage indicator that is equal to one if the company is in the expansion or later stage, and equal to zero otherwise. These variables are common throughout the literature, since VC investment decisions are influenced by the industry of the company as well as how close they are to generating a consistent revenue stream. I also include an indicator that is equal to one if the company is business-facing rather than consumer-facing. Finally, I use the log of the initial investment in terms of 2012 USD.

VC Characteristics. VC characteristics are much more abundant in the VentureXpert Database, which allows for a more detailed set of variables. In this section, I will discuss how the variables are computed as well as their importance to portfolio companies when choosing a VC. I use experience characteristics similar to those from Sørensen (2007). The main experience variable of interest is the number of first round lead investments made by a VC prior to the time \( t \). This experience measure is calculated by beginning with investments made in 1975. Thus, the characteristic is written \( \text{Experience} = \sum_{\tau=1}^{t-1} |\mu_{\tau}(j)| \), where \( |\mu_{\tau}(j)| \) represents the number of lead investments made by VC \( j \) at time \( \tau \). VC experience is valued by companies, while also being an important indicator of a VC’s ability to guide a company to a positive outcome. The number of investments made by a VC in a given year are not included in the measure for that year, because those investments are not visible to the companies at the time of investment. As a result, they should not influence a company’s valuation of a VC. The importance of this variable in the matching market is of central importance to the arguments in Sørensen (2007), whose results justify its inclusion in the model. I also use the specialization measure created in Gompers et al. (2009), that is the HHI for the number of investments made by a VC based on company industry. Instead of the three major industries used as company characteristics, I choose a more specific definition that gives 10 different industries. The company industries are: Biotechnology, Communications and Media, Computer Hardware, Computer Software and Services, Consumer Related, Industrial/Energy, Internet Specific, Medical/Health, Semiconductors/Other Electronics, and Other Products. This measure considers the total number of investments made by a VC based on company industry. Instead of the three major industries used as company characteristics, I choose a more specific definition that gives 10 different industries. The company industries are: Biotechnology, Communications and Media, Computer Hardware, Computer Software and Services, Consumer Related, Industrial/Energy, Internet Specific, Medical/Health, Semiconductors/Other Electronics, and Other Products. This measure considers the total number of investments made by a VC in each industry, \( m \), which is denoted by \( \text{tot}_{-\text{exp}}_{jmt} \). The measure is updated every year similarly to the experience variable in that the HHI at time \( t \) represents investments made prior to \( t \), so that it is written:

\[
HHI_{jt} = \sum_{m=1}^{10} s_{jmt}^2
\]
where
\[
s_{jmt} = \frac{\text{tot\_exp}_{jmt}}{\text{tot\_exp}_{jt}} = \frac{\sum_{\tau=1}^{t-1} \text{tot\_exp}_{j\tau}}{\sum_{\tau=1}^{t-1} \text{tot\_exp}_{j\tau}}
\] (17)

The variable offers another dimension by which companies are able to judge to proficiency of the VC firm. A VC may be involved in a large number of investments, but does not specialize in a particular industry. Thus, they may not be as capable of guiding a portfolio company to a positive outcome relative to a highly specialize VC.

Finally, I create two characteristics that measure VC reputation. Multiple papers find that VCs use IPOs and even acquisitions as a form of reputation building in order to attract companies with higher potential profits (Chaplinsky and Gupta-Mukherjee, 2010,WP; Shu et al., 2011; Nahata, 2008). In order to capture this effect, I use two measures: one of prior IPO success and the other of prior acquisition success. I will focus on the definition for prior IPO success for now, but the formulation of prior acquisition success is similar. The measure is the percent of investments that end with IPOs prior to a time \( t \). The measure is written as:
\[
\text{IPO Rate}_{jt} = \frac{\sum_{\tau=1}^{t-1} \sum_{i \in \mu_{\tau}(j)} d_{ipo}^{ij\tau}}{\sum_{\tau=1}^{t-1} |\mu_{\tau}(j)|}
\] (18)

where \( d_{ipo}^{ij\tau} \) is an indicator equal to one if the investment by VC \( j \) in company \( i \) at time \( \tau \) resulted in an IPO, and the IPO date was before time \( t \). Thus, \( \text{IPO Rate}_{jt} \) represents the fraction of VC \( j \)'s lead investments that resulted in IPOs prior to time \( t \). The requirement that the IPO date be before time \( t \) is vital to the definition of the variable as a measure of reputation, because at time \( t \) a company must be able to look at VC \( j \)'s investments and calculate their IPO rate.

**Syndication Contracts.** The contract variable is created to account for both the network position of the lead VC as well as the capital constraints it faces. The measure is given by:
\[
\text{Net. Adjusted Capital} \equiv \mathcal{N}_{jt(5)} = \frac{\ln(\text{Available\_Capital}_{jt})}{1 + \text{norm\_closeness}_{jt(5)}} = \frac{\Lambda_{jt}}{1 + \mathcal{C}_{jt(5)}}
\] (19)

where \( \Lambda_{jt} \) is \( \ln(\text{Available\_Capital}_{jt}) \), and \( \mathcal{C}_{jt(5)} \) represents the normalized closeness measure of the investment network, \( \text{norm\_closeness}_{jt(5)} \). The Net. Adjusted Capital (\( \mathcal{N}_{jt(5)} \)) is a measure of a VC’s potential spending power weighted by its network position. The network position of the lead VC is an important determinant in a company’s preference for a syndicated contract. If a VC is well connected, then it is easier for the VC to bring in other high quality VCs that add value to the company. Additionally, a VC may be more inclined to syndicate if it has more connections, so a VC’s network position can influence its own preferences. In the VC industry, an edge in the graph represents a link between to VCs investing in the same company, while each constitutes a node. As in Hochberg et al. (2007), I define the network for year \( t \) as the five years prior, \( t(5) \), because the structure of the VC industry is constantly changing. For example, a connection between two VCs from many years ago may not still represent a viable relationship, and should not be counted when determining a VC’s network position. I choose to focus on the closeness measure of network centrality. This measure is created using the undirected \( t(5) \) network comprised of investments made by all VCs.\(^{15}\) I normalize this closeness measure within a given matching

\(^{15}\)In an undirected network, a path from VC \( j \) to VC \( k \) is equivalent to a path from \( k \) to \( j \). In a directed network, the two paths are treated as different. If \( j \) brings in \( k \) as a co-investor, it is considered an outgoing edge for \( j \) and an incoming edge for \( k \).
market, with the measure given below:

\[ \text{norm\_closeness}_{j(t)} \equiv C_{j(t)} = U \left[ \frac{1}{D_j} \left( \frac{A(j)}{J_{(5)} - 1} \right)^2 \right], \quad D_j = \sum_{k \in A(j)} d_{j,k} \]

(20)

where \( U[\cdot] \) represents a min-max normalization, \( A(j) \) is the set of reachable nodes for VC \( j \), \( J_{(5)} \) is the total number of VCs in the network, \( D_j \) is the sum of the distance from VC \( j \) to all other VCs, and \( d_{j,k} \) is the distance between VC \( j \) and VC \( k \). The distance between two nodes is defined as the shortest number of edges between the two nodes. In the context of the VC network, the distance between two VCs \( j \) and \( k \) is equal to 2 if they both co-invested with some VC \( j' \), but not with each other. Figure 2 depicts an undirected network consisting of five VCs and five companies. In the network, an edge between two VCs represents the company in whom the two VCs co-invested. For example, \( VC_A \) and \( VC_D \) co-invest in Company 1, but they also co-invest with \( VC_B \) in Company 2. As an example, I compute the closeness measure for \( VC_A \). In this case, the set of reachable nodes and the total number of VCs minus one are both four, so the closeness measure simplifies to \( \frac{1}{4} \). Since \( VC_A \) directly co-invests with \( VC_B \) and \( VC_D \), the distance between each is equal to one. The distance between \( VC_A \) and \( VC_C \) is equal to two, because they directly co-invest with a common VC. Finally, the distance between \( VC_A \) and \( VC_E \) is equal to three. Thus, the total distance for \( VC_A \) is \( D_A = d_{A,B} + d_{A,D} + d_{A,C} + d_{A,E} = 1 + 1 + 2 + 3 = 7 \), which implies the closeness for \( VC_A \) is \( 1/7 \).

In addition to network position, some of the main documented reasons for VCs choosing to syndicate are to relax capital constraints, mitigate risk, and diversify their investments, (Deli and Santhanakrishnan, 2010; Hopp and Rieder, 2011). The measure I create is the log of all lead investments during the same year by a given VC, which I refer to as \( \log(\text{Available Capital}_{jt}) \), or \( \Lambda_{jt} \). This captures a VC’s capital constraints, since VCs that are capable of investing large amounts are more likely to be less capital constrained. The combined measure used in the model captures the tradeoff between capital constraints and network position. If a VC has ample capital and very few connections to other VCs, then it is unlikely to offer a company a syndicated contract. On the other hand, a capital

Figure 2: Undirected Investment Network. Notes: Each edge between VCs represents the company in which both VCs co-invested.
constrained VC with a similar network position is more likely to offer a syndicated contract simply due to its inability to finance the company as a solo investor. Thus, a VC is more likely to offer a syndicated contract when the measure is small and less likely when it is large.

6. Empirical Findings

In this section, I provide outcome estimates for the standard Probit model. Then, I discuss the presence of sorting. Next, I present the results for the structural model. Afterward, I examine the bias caused by sorting.

The relationship between syndication and a company IPO is examined by estimating a Probit model (see Table 4). There are a couple notable results in the table. First and foremost, syndication increases the likelihood that a company achieves an IPO, and this relationship is robust to VC fixed-effects. Second, unlike in Sørensen (2007), VC experience is relatively unimportant in determining a company’s IPO probability. Thus, the standard Probit model indicates that syndication is far more important in determining a company’s outcome than VC experience, which signifies a change in the VC industry. Additionally, it is very clear that companies in the Medical/Health/Life Science industry are far more likely to exit by IPO than Non-High Tech companies, while IT companies are not any more likely to achieve IPOs than Non-High Tech companies. Since it is unlikely that Medical/Health/Life Science companies are systematically higher quality than companies in other industries, the positive and significant coefficient points to IPOs being the preferred exit strategy for companies in this industry.

6.1. Sorting

There are two types of sorting that can induce a bias in the outcome estimates. One involves sorting along observable variables, while the other is sorting on unobservables. The presence of sorting on observable characteristics is simple to confirm. A robust and significant relationship between characteristics on each side of the market indicates that sorting may lead to biased outcome estimates. Basic regressions of characteristics from one side on characteristics of the other can provide the necessary intuition to determine the presence of sorting, as well as the variables on which sorting is most intense. Since my main interest is the sorting into syndicates, I focus on the regressions of syndication on VC and company variables. I present these assortativity regressions in Table 5. Then I turn to the preference estimates from the Bayesian model, which are given in Table 6, in order to further examine the presence of sorting.

6.1.1 Sorting into Syndicates.

The relationship between syndication and agent characteristics provides an idea as to which factors influence the syndication decision of a matched pair. This relationship is captured in the Probit regressions of syndication on VC and company covariates presented in Table 5. The robust, negative relationship between syndication and Net. Adjusted Capital indicates that VCs with more available capital per connection are less likely to syndicate. This implies that VCs seek to syndicate less often when they have more available capital, while a more prominent network position makes syndication more likely. The other
Table 4: Probit IPO Probability

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Syndication</td>
<td>0.1857**</td>
<td>0.2019**</td>
</tr>
<tr>
<td></td>
<td>(0.0918)</td>
<td>(0.0963)</td>
</tr>
<tr>
<td>Experience</td>
<td>0.0007</td>
<td>-0.0029</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.0058)</td>
</tr>
<tr>
<td>IPO Rate</td>
<td>0.5041</td>
<td>-0.5239</td>
</tr>
<tr>
<td></td>
<td>(0.3814)</td>
<td>(0.6453)</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>-0.1112</td>
<td>0.2975</td>
</tr>
<tr>
<td></td>
<td>(0.2469)</td>
<td>(0.4382)</td>
</tr>
<tr>
<td>VC Industry HHI</td>
<td>-0.0049</td>
<td>0.0481</td>
</tr>
<tr>
<td></td>
<td>(0.3930)</td>
<td>(0.6866)</td>
</tr>
<tr>
<td>Stage</td>
<td>0.0349</td>
<td>0.0592</td>
</tr>
<tr>
<td></td>
<td>(0.1111)</td>
<td>(0.1182)</td>
</tr>
<tr>
<td>IT</td>
<td>0.1265</td>
<td>0.0941</td>
</tr>
<tr>
<td></td>
<td>(0.1585)</td>
<td>(0.1627)</td>
</tr>
<tr>
<td>Medical/Health/Life Science</td>
<td>1.0079***</td>
<td>1.0651***</td>
</tr>
<tr>
<td></td>
<td>(0.1748)</td>
<td>(0.1837)</td>
</tr>
<tr>
<td>Business-Facing</td>
<td>0.1684*</td>
<td>0.1717*</td>
</tr>
<tr>
<td></td>
<td>(0.0874)</td>
<td>(0.0921)</td>
</tr>
<tr>
<td>ln(Initial Investment)</td>
<td>0.1963***</td>
<td>0.1582***</td>
</tr>
<tr>
<td></td>
<td>(0.0411)</td>
<td>(0.0509)</td>
</tr>
<tr>
<td>ln(Distance)</td>
<td>-0.0255</td>
<td>-0.0194</td>
</tr>
<tr>
<td></td>
<td>(0.0169)</td>
<td>(0.0190)</td>
</tr>
</tbody>
</table>

FE                     | mt           | jmt          |
Observations            | 2,665        | 2,665        |

Standard errors in parentheses. * p < 0.1, ** p < 0.05, *** p < 0.01

Notes: This table presents estimates from two Probit models. The dependent variable is IPO. The estimates presented are the raw coefficients from the model and not marginal effects. The fixed-effects in the model are dummy variables, so Column (1) contains dummy variables for the two main industries IT and Medical/Health/Life Science, and it also contains year dummies that are not included in the table. Column (2) adds VC dummy variables. The VCs that never take a company public are grouped into a single group, while the remaining VCs have their own fixed effects. This is done to avoid dropping the investments by these VCs.
noteworthy result is the significance of the negative coefficient for VC experience after introducing VC fixed-effects. The negative relationship between syndication and VC experience provides support for the selection motivation for syndication.

Unsurprisingly, early stage companies are more likely to syndicate than late stage companies. It can be argued that this result supports both the selection motivation and the value-added motivation, since early stage companies stand to gain more from additional advice while being significantly riskier investments than their late stage counterparts.

6.1.2 Bayesian Model Results.

Here I discuss the results from the Bayesian structural model selection equation presented in Table 6. I leave an examination of the Bayesian outcome estimates for Section 6.2. First, note that the mean preference for syndication for both VCs and companies is positive. The implication for companies is that they will prefer syndication over non-syndication with the same VC. However, this is not necessarily the case for VCs. The negative coefficient for Net. Adjusted Capital indicates that the benefit of syndication is decreasing as the Net. Adjusted Capital increases. Thus, for VCs with sufficiently large Net. Adjusted Capital, non-syndication will be the preferred contract. The impact that these preferences have on outcome estimates will become clear when comparing the Bayesian outcome equation results to the standard Probit. One interesting result is that despite VC experience no longer influencing IPO probability, companies still prefer more experienced VCs. Another result that requires explanation is VC preference for early stage companies. Since late stage companies tend to be less risky investments, one would expect VCs to prefer these companies. However, the preference estimates are only for the first investment in a company. Thus, VCs searching for companies in this type of market may prefer early stage companies, as these companies offer larger return on investment. Finally, the Covariance coefficients from the model, $\delta$ and $\kappa$, show the degree of sorting on unobservable characteristics. While $\kappa$ is not significantly different from zero, $\delta$ is statistically significant at the 10% level. This implies that matching between VCs and companies influences company outcomes. I discuss the importance of sorting on the unobservables in more detail in Section 6.2.3.

6.2. Effect of Sorting

Now I examine the impact of sorting on the outcome equation estimates, and compute the various forces discussed in the introduction. First, consider the outcome equation results from the Bayesian model presented in Table 6. The average increase in probability of an IPO when an investment is syndicated, also referred to as the VA, is 2.07%. Although this effect may not seem substantial, recall that only 6.5% of all investments result in IPOs, so syndicating an investment increases this probability by more than 30%. Thus, we see that there is value in syndicating an investment that does not disappear when accounting for sorting. In fact, when comparing the results from the Bayesian model to the Probit model with fixed-effects, the difference is small. The Biased VA from the Probit model gives an average increase in probability of an IPO of 2.01%. Consequently, the SE is only -.06%, which implies a downward bias of only 2.74%. There are multiple reasons for the negligible bias, which are related to the preferences for syndication on both sides of the market. It is also important to caution against assuming a standard Probit with VC fixed-effects will
Table 5: Probit Syndication Probability

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net. Adjusted Capital</td>
<td>-0.2279***</td>
<td>-0.1812***</td>
</tr>
<tr>
<td></td>
<td>(0.0417)</td>
<td>(0.0484)</td>
</tr>
<tr>
<td>Experience</td>
<td>-0.0004</td>
<td>-0.0075***</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>IPO Rate</td>
<td>0.3223</td>
<td>0.0833</td>
</tr>
<tr>
<td></td>
<td>(0.3017)</td>
<td>(0.4040)</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>0.0050</td>
<td>0.3983**</td>
</tr>
<tr>
<td></td>
<td>(0.1305)</td>
<td>(0.1587)</td>
</tr>
<tr>
<td>VC Industry HHI</td>
<td>0.8762***</td>
<td>0.0493</td>
</tr>
<tr>
<td></td>
<td>(0.2455)</td>
<td>(0.3189)</td>
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<tr>
<td>Stage</td>
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<td>-0.4276***</td>
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<tr>
<td></td>
<td>(0.0759)</td>
<td>(0.0811)</td>
</tr>
<tr>
<td>IT</td>
<td>0.2554***</td>
<td>0.1817*</td>
</tr>
<tr>
<td></td>
<td>(0.0901)</td>
<td>(0.0971)</td>
</tr>
<tr>
<td>Medical/Health/Life Science</td>
<td>0.0944</td>
<td>-0.0570</td>
</tr>
<tr>
<td></td>
<td>(0.1159)</td>
<td>(0.1297)</td>
</tr>
<tr>
<td>Business-Facing</td>
<td>-0.0440</td>
<td>-0.0481</td>
</tr>
<tr>
<td></td>
<td>(0.0548)</td>
<td>(0.0571)</td>
</tr>
<tr>
<td>ln(Initial Investment)</td>
<td>0.4165***</td>
<td>0.4153***</td>
</tr>
<tr>
<td></td>
<td>(0.0254)</td>
<td>(0.0315)</td>
</tr>
<tr>
<td>ln(Distance)</td>
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<td>-0.0052</td>
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<tr>
<td></td>
<td>(0.0104)</td>
<td>(0.0118)</td>
</tr>
<tr>
<td>FE</td>
<td>mt</td>
<td>jmt</td>
</tr>
<tr>
<td>Observations</td>
<td>2,665</td>
<td>2,665</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. * p < 0.1, ** p < 0.05, *** p < 0.01

Notes: This table presents estimates from two Probit models. The dependent variable is Syndication. The estimates presented are the raw coefficients from the model and not marginal effects. The fixed-effects in the model are dummy variables, so Column (1) contains dummy variables for the two main industries IT and Medical/Health/Life Science, and it also contains year dummies that are not included in the table. Column (2) adds VC dummy variables. The VCs that never syndicate an investment are grouped into a single group, while the remaining VCs have their own fixed effects. This is done to avoid dropping the investments by these VCs.
Table 6: Bayesian Model Outcome Equation

<table>
<thead>
<tr>
<th>Outcome Equation</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-3.2597***</td>
<td>( 0.4312)</td>
</tr>
<tr>
<td>ln(Distance)</td>
<td>-0.0235</td>
<td>( 0.0168)</td>
</tr>
<tr>
<td>Stage</td>
<td>0.0557</td>
<td>( 0.1105)</td>
</tr>
<tr>
<td>IT</td>
<td>0.0857</td>
<td>( 0.1603)</td>
</tr>
<tr>
<td>Medical</td>
<td>1.0138***</td>
<td>( 0.1774)</td>
</tr>
<tr>
<td>ln(Initial Investment)</td>
<td>0.1953***</td>
<td>( 0.0401)</td>
</tr>
<tr>
<td>Business-Facing</td>
<td>0.1943**</td>
<td>( 0.0869)</td>
</tr>
<tr>
<td>Experience</td>
<td>0.0001</td>
<td>( 0.0007)</td>
</tr>
<tr>
<td>IPO Rate</td>
<td>0.4226</td>
<td>( 0.3627)</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>-0.3259</td>
<td>( 0.2296)</td>
</tr>
<tr>
<td>$HHI_{jt}$</td>
<td>-0.3986</td>
<td>( 0.3906)</td>
</tr>
<tr>
<td>Syndication</td>
<td>0.1854**</td>
<td>( 0.0903)</td>
</tr>
</tbody>
</table>

Selection Equation

VC Preferences:
- Stage: -0.0595*** ( 0.0142)
- IT: 0.3765*** ( 0.0158)
- Medical: 0.4206*** ( 0.0228)
- ln(Initial Investment): 0.1146*** ( 0.0027)
- Business-Facing: 0.0197* ( 0.0108)
- Syndication: 0.1990*** ( 0.0767)
- Net. Adjusted Capital ($N_{jt(5)}$): -0.0292** ( 0.0134)

Company Preferences:
- Experience: 0.0027*** ( 0.0007)
- IPO Rate: 1.1172*** ( 0.3957)
- Acquisition Rate: 0.9085*** ( 0.1426)
- $HHI_{jt}$: 1.5137*** ( 0.2981)
- Syndication: 0.2141** ( 0.0853)

Covariances:
- $\delta$: -0.0323* ( 0.0232)
- $\kappa$: -0.0235 ( 0.0745)

N: 2,665

Notes: This table presents estimates from the structural model. The dependent variable in the outcome equation is IPO. The estimates presented for both the outcome and selection equations are the raw coefficients from the model and not marginal effects. There are two dependent variables in the selection equation. One is the VC latent utility and the other is the company latent utility. For VC preferences, a positive coefficient indicates that an increase in the covariate will increase the probability that the VC prefers the company to an otherwise identical company. For company preferences, a positive coefficient indicates that an increase in the covariate will increase the probability that the company prefers the VC to an otherwise identical VC. The covariance, $\delta$, is bounded from above by 0, so corresponding p-value is one-sided. The estimates are based on the remaining 5,667 simulations of the posterior distribution that remain after discarding initial draws for burn-in and thinning.

Standard errors in parentheses. * (p<0.1), ** (p<0.05), *** (p<0.01)
always produce estimates that are reasonably close to a model that accounts for sorting. I discuss both of these topics in detail in the remainder of this section.

6.2.1 Forces of Syndication

The relationship between Company SSE, VC SSE, and the three motivations for syndication is key to understanding the sign and magnitude of the estimated bias. I start by discussing the Company SSE and VC SSE. Recall that each of these terms refers to the use of a syndicated contract as a selection mechanism. In the case of companies, this means accepting a non-syndicated contract from a high quality VC when a better company is not willing to do so. On the other hand, a VC can use a syndicated contract to lure a company away from a better VC who is not willing to offer the company a syndicated contract. The better VC is faced with a choice between offering a syndicated contract in order to keep the company, or turning their attention to another company. Each of these effects can be viewed as a way of measuring how agents on one side of the market can match with better agents on the other side of the market by leveraging their preferences over contracts. The purest method of quantifying this is by computing the change in utility for each side of the market as a result. For example, I measure the VC SSE as the utility a VC gives up when forced into offering a syndicated contract. More specifically, it is the difference between the utility a VC receives under each contract when it prefers non-syndication, but offers a syndicated contract in order to keep the company. The Company SSE is the utility gained by the company at the expense of the VC when it prefers a non-syndicated contract. Although it is not possible to compare magnitudes of utilities between companies and VCs, I can compare the percent change in utility each side receives. The VC SSE is computed as follows, and the Company SSE is computed in a similar manner:

$$\text{VC SSE} = \frac{1}{T} \sum_{t} \frac{1}{|\chi_t|} \sum_{i \in \chi_t} \sum_{s} \frac{v_{i \mu(i)s}^s - v_{i \mu(i)s}^n}{v_{i \mu(i)s}^n}$$

where $\chi_t$ is the set of syndicated matches for which the VC prefers non-syndication in market $t$. The VC SSE is -24.62%. This implies that on average, VCs that choose to syndicate when they prefer non-syndication, but offers a syndicated contract in order to keep the company. The Company SSE is the utility gained by the company at the expense of the VC when it prefers a non-syndicated contract. Although it is not possible to compare magnitudes of utilities between companies and VCs, I can compare the percent change in utility each side receives. The VC SSE is computed as follows, and the Company SSE is computed in a similar manner:

In order to offer further intuition, I use company preference for VC experience to highlight the Company SSE. Suppose a company receives a syndicated contract offer from VC $j$. Additionally, the company receives a non-syndicated contract offer from VC $j'$, who only differs from VC $j$ in experience. In order for the company to be indifferent between the two VCs, it must be that $exp_j \beta^{exp} + \beta^c = exp_{j'} \beta^{exp}$, where $\beta^c$ is the coefficient for mean syndication preference for companies. This equality can be rearranged to obtain

$$\frac{\beta^c}{\beta^{exp}} = \frac{exp_{j'} - exp_j}{80.45}$$

This implies that a VC offering a company a syndicated contract has the same probability of the company accepting as a VC with 80.45 more investments of experience offering a non-syndicated contract. This is staggering considering the mean VC experience level in the sample is 75.63. While it is possible to compute a similar measure to highlight the VC SSE, no company characteristic offers as clear a picture as VC experience. Thus, I turn to the motives of syndication to further explore the VC SSE.

6.2.2 Motives for Syndication

The three motives for syndication allow for even deeper insight into the VC SSE, which of course helps to understand the SE. Each measure is based upon the difference in VC
utility across contracts. First, I consider the finance-related motive, which I denote as \( FRSE \). I compute this as the change in the probability that a VC prefers syndication to non-syndication given a change in their capital constraints. In the context of the model, the capital constraint of a VC \( j \) in market \( t \) is \( \Lambda_{jt} \). The \( FRSE \) for a particular match is defined as:

\[
FRSE_{ijt,h} = \frac{\partial \Pr(v^s_{ijt,h} > v^n_{ijt,h})}{\partial \Lambda_{jt}} = \frac{\partial \Pr(\alpha_{0,h} + N_{ijt}^{(5)}\alpha_{1,h} > \varepsilon_{ijt,h}^n - \varepsilon_{ijt,h}^s)}{\partial \Lambda_{jt}}
\]

(22)

where \( h \) denotes the iteration of the Gibbs sampler. The second equality in the first line of (22) comes from the utilities under each contract differing by the error terms and the contract specific terms. We get the final equality from (19). The measure for the entire sample is the average change across all matches is:

\[
FRSE = \frac{1}{N} \sum_{t} \sum_{i \in I_t} \frac{1}{H} \sum_{h} FRSE_{ijt,h}
\]

(23)

where \( N \) is the total number of companies in the sample. This is computed as -.0049, which indicates that given a unit change in \( \Lambda_{jt} \) the probability that a VC prefers a syndicated contract decreases on average by .49%. This result provides support for the theory that VCs are more likely to syndicate investments when they face more restrictive capital constraints. In terms of the effect on selection mechanism, if less constrained VCs are viewed as being better able to guide a company to an IPO, then their companies would have a higher likelihood of going public than more constrained VCs. When observed without any knowledge of the matching market, this would seem like non-syndicated companies have a higher likelihood of achieving an IPO. Thus, the \( FRSE \) acts as a negative force on the estimates from a standard Probit. In order to illustrate the \( FRSE \), I plot the probability advantage of syndicating for each decile of available capital in Panel A of Figure 3. The probability advantage is the probability that a syndicated contract is preferred minus the probability that a non-syndicated contract is preferred. For an individual match this is given by:

\[
Pr(v^s_{ijt} > v^n_{ijt}) - Pr(v^n_{ijt} > v^s_{ijt}) = 2 \times Pr(\alpha_{0,h} + N_{ijt}^{(5)}\alpha_{1,h} > \varepsilon_{ijt,h}^n - \varepsilon_{ijt,h}^s) - 1
\]

(24)

Since this is closely linked to the \( FRSE \), we see that the probability advantage of syndicating is decreasing as the decile of available capital increases.

The next motive I consider is the value-added motivation, or \( VA_{VC} \), which can be evaluated using the preference estimates in Table 6. The measure is simply the added utility a VC receives from syndication, which is \( \alpha_{0,h} + N_{ijt}^{(5)}\alpha_{1,h} \). Given the signs for each of the coefficients, the value-added from a syndicated contract diminishes as a VC’s capital constraints are loosened, but increases as they become more connected. Both of these effects match the results from previous papers, and offer further insight into the VC SSE. Since it is possible for the \( VA_{VC} \) to be positive or negative, the impact on the direction and magnitude of the bias depends heavily on how the unobserved quality of VCs is related to both their capital constraints and their network position. One might expect better VCs to be less capital constrained and have a better network position, which would result in the two components

\[16\] A unit change in \( \Lambda_{jt} \) is equivalent to a change of 1000 \( \cdot \) 6 in 2012 USD.
of the $V_A V_C$ working in opposition. Consequently, even though there is substantial sorting, the $V_A V_C$ may end up having a negligible effect on the bias. A visual representation of the $V_A V_C$ is plotted in Panel C of Figure 3. This shows the probability advantage of syndication for each decile of the contract variable. Since this variable is included in the calculation of the probability advantage, the plot is much smoother than the other two.

![Panel A: Finance-Related Motivation](image1.png)

![Panel B: Selection Motivation](image2.png)

![Panel C: Value-Added Motivation](image3.png)

Figure 3: Syndication Motivations by Decile. Notes: (i) Panel A plots the probability advantage for deciles of $\Lambda_{jt}$. (ii) Panel B plots the probability advantage for deciles of experience. (iii) Panel C plots the probability advantage for deciles of $Net. Adjusted Capital, (N_{jt})$.

Finally, I turn to the selection motive, or pure selection effect ($PSE$). Since this force is based upon a VC’s screening ability, which is unobservable to the econometrician; it is difficult to compute directly. In theory, a VC’s experience should provide the econometrician with some information about its screening ability (Cassamatta and Haritchabalet, 2007). Unfortunately, in the case of my model, VC experience does not directly enter into VC preferences over contracts. Thus, I examine how the probability that a VC chooses a syndicated contract changes as VC experience increases through numerical methods. I do this by calculating the probability advantage of a syndicated contract over a non-syndicated contract for each decile of VC experience. Under the assumption that more experienced VCs possess better screening ability, and VCs with better individual screening ability are less likely to syndicate an investment in order to obtain a second opinion; the $PSE$ should be decreasing as VC experience increases. I plot the probability advantage for syndicated contracts across deciles of VC experience in Panel B of Figure 3, in order to illustrate the $PSE$. Through the first three deciles, the probability advantage of syndication is increasing, but begins to decline after the 30th percentile. While it may seem counterintuitive for the probability advantage to increase at all, there is a simple explanation for the increase at lower deciles of VC experience. Recall from Lerner (1994), that VCs syndicating early round investments are likely to invest with VCs of similar experience. Consequently, for the least experienced VCs, the value of a second opinion is only as good as a similarly inexperienced VC. This leads to the value of syndication increasing up to a certain level of VC experience, after which the benefit of a second opinion begins to decline. The decreasing probability advantage occurs as VCs get more experienced, because they are sufficiently capable at screening to discern the value of a potential investment without the help of another VC.

In examining the three motivations of syndication, I have rationalized the negative VC SSE. I have also demonstrated the potential for these forces to work in opposition, especially in the case of the $V_A V_C$. The complicated and imperfect relationship between all three of these measures shows that it is not guaranteed for the SE to be negligible. I illustrate this
6.2.3 Sorting Over Time

In this section I caution against the false conclusion that a Probit with VC fixed-effects will always produce similar results to the Bayesian model. I begin by examining the Biased VA and VA for each year, which are plotted in Panel A of Figure 4.

Surprisingly, the Probit model overestimates the VA from syndication for each year be-
fore 2010 despite the overall model marginally underestimating the VA. This discrepancy is clearly due to the large drop in the average partial effect in the Probit model after 2009, at which point the Probit model vastly underestimates the VA. In Panel B, we can see how these changes across years impact the bias, with it hovering between as low as 5% to nearly 40% for years 2004-2009. However, for 2010-2012 the bias gets lower than -40%. It is already evident from Panels A and B that the Probit model will not always produce estimates that are reasonably close to the Bayesian model. Additionally, we see that the APE for both models is declining over time, however, the APE for the Probit is far more volatile than the Bayesian model. This extra volatility can be attributed to changes in sorting along VC and company unobservables over time. Since the Bayesian model controls for sorting along the unobservables, the APE is not affected by their volatility. In Panel C of Figure 4, the interaction between the estimated covariance coefficient and the mean VC error terms under each contract are plotted for each year in the sample. These are the terms that enter the outcome equation, and influence the IPO probability. The impact of the error terms on the APE for the Probit model is most noticeable for 2010-2012, where we see a large decrease in the syndicated contract error term, $\hat{\delta}_s$. In terms of the bias, the difference in the error terms across contracts is especially important. This is depicted in Panel D with $\Delta \hat{\delta}_s \equiv \hat{\delta}_s - \hat{\delta}_n$. Except for a dip in 2005, the bias is positive when the difference in error terms is positive. Conversely, when the difference is negative for 2010-2012, the bias is negative. A cursory check of the correlation between the error terms and the sample year confirms the conclusions drawn from Panels C and D. These are $corr(t, \hat{\delta}_s) = -0.119$ for the syndicated contract error terms and $corr(t, \hat{\delta}_n) = -0.0586$ for the non-syndicated contract error terms. We see that both of these effects become more negative over time, with the syndicated contract errors decreasing at a faster rate than the non-syndicated errors. The implication for the yearly APE is that the Biased VA decreases more quickly than the VA, which eventually results in the bias changing from positive to negative over the course of the sample.

7. Counterfactual Exercises

In this section, I conduct a counterfactual exercise that illustrates the power of the two-sided matching model as a tool for policy analysis. I examine the effect of changes in the long-term corporate capital gains tax. Only recently has the academic literature begun to address the impact of the capital gains tax on the VC industry. On the theoretical side, (Keuschnigg and Nielsen, 2003, 2004a,b), have shown increases in capital gains taxation reduce industry activity leading to a reduction in VC and entrepreneur welfare. Additionally, they find that a reduction in the capital gains tax rate is more effective as a policy tool than a subsidy. On the empirical side, Achleitner et al. (2011) provide evidence that increases in the capital gains tax rate reduces the number of initial investments made by VCs.

Given the results from these two strands of literature, I develop a counterfactual experiment to examine the influence of changes in the capital gains tax rate on the industry through its effect on the matching market. Since the tax rate has been directly linked to number of initial investments, I focus on the change in VC portfolio size in response to changes in the capital gains tax rate. However, this approach comes with a caveat. Ideally, one would use a model in which the portfolio size responds endogenously, which would require the tax rate to impact portfolio size through VC preferences. Consequently, a VC may find it favorable to adjust portfolio size based on the cost of financing each company.

\footnote{I estimate two separate Bayesian models for the periods from 2004-2006 and 2007-2012, in order to ensure that the change in bias is not dependent upon the estimates for the entire sample. I find that the bias for 2004-2006 is 18.72%, while the bias for 2007-2012 is -30.93%. Both of these results reflect the bias depicted in Panel B. I leave the full tables from these two models for the online appendix.}
This would imply that company-contract pairs are no longer substitutes, as the value of each potential investment is no longer independent of the other companies in a VC’s portfolio. A well known feature of matching models is that when at least one side of the market exhibits preferences with complementarities, it is not guaranteed that an equilibrium is unique or even that one exists. Rather than dealing with this type of issue directly, (see Vissing (2018,WP); Uetake and Watanabe (2020,WP) for empirical models addressing this problem), I opt for a reduced form approach to generating a change in portfolio sizes. More specifically, I estimate a simple regression model of portfolio size on the tax rate and a variety of controls including a one year lag of portfolio size. The estimated equation is given below:

\[
Portfolio\ Size_{jt} = P(\tau_{cg}^t, \Lambda_{jt})\beta_p^j + controls_{jt}\beta_c^j + \epsilon_{jt}
\]  

where \(P(\tau_{cg}^t, \Lambda_{jt})\) is a polynomial matrix of the capital gains tax, \(\tau_{cg}\), VCs’ available capital, \(\Lambda_{jt}\), and the interaction of the two. The polynomial of the interaction term is especially important as it allows the impact of the tax to be different across VCs based on their capital constraints. Since the tax rate changes are infrequent from 2004 to 2012, I expand the sample to include VC-year observations from 1975 to 2018. This provides sufficient variation to produce predicted portfolio sizes that are not overly sensitive to changes in the tax rate. Finally, I exclude outliers in portfolio size over this period to reduce their influence on the predicted values. Since I am more interested in the estimation of this equation as a method for generating predicted portfolio size, I focus on the explanatory power rather than the coefficients. The preferred specification I use has an R-squared of .905, indicating that the predicted values should closely resemble the observed values.\(^\text{18}\) I use the estimates from the reduced form regression to compute the predicted portfolio size given the observed tax rates. Once I obtain the predicted values, I round these to the nearest integer to obtain the portfolio sizes I use in the simulation procedure, which I denote \(\hat{q}_{jt}\) for a given VC-year.

I leave a more thorough discussion of the estimation of the reduced form equation for the online appendix.

\[^{18}\text{I leave a more thorough discussion of the estimation of the reduced form equation for the online appendix.}\]

Table 7: Predicted Portfolio Size Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Obs</th>
<th>Total Capacity</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>486</td>
<td>2,665</td>
<td>5.48</td>
<td>3.73</td>
<td>2</td>
<td>21</td>
<td>4</td>
</tr>
<tr>
<td>Baseline</td>
<td>466</td>
<td>2,237</td>
<td>4.99</td>
<td>2.86</td>
<td>1</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>(\tau_{cg} = 10%)</td>
<td>486</td>
<td>3,822</td>
<td>7.86</td>
<td>3.05</td>
<td>2</td>
<td>26</td>
<td>7</td>
</tr>
<tr>
<td>(\tau_{cg} = 20%)</td>
<td>463</td>
<td>2,323</td>
<td>5.02</td>
<td>2.80</td>
<td>1</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>(\tau_{cg} = 35%)</td>
<td>263</td>
<td>797</td>
<td>3.03</td>
<td>2.22</td>
<td>1</td>
<td>13</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes: The statistics in the table are computed only for the main sample for the paper (i.e. 2004-2012) after rounding to integers and dropping values less than or equal to zero. The observed statistics are for the actual portfolio sizes from the data. The baseline statistics are the predicted portfolio sizes using the tax rates observed in the data. The remaining statistics are for the different tax rates applied uniformly to the sample.
composition of portfolio sizes for VCs that remain in the market. We can see that while the median portfolio size is inversely related to the tax rate, the same is also true for the standard deviation. Thus, portfolio sizes not only shrink, but the distribution also becomes less dispersed.

7.1. Market Simulation

I use the predicted portfolio sizes under each tax rate I simulate matching markets using the parameter estimates from the Bayesian model and the observed data. This involves computing the deterministic portion of the VC and company utilities, taking $S$ draws of the errors terms, $\xi$ and $\varepsilon$, and simulating the matching markets for each draw. The utilities for a given simulation draw are:

$$
u^{c}_{jits} = Z^{c}_{jt} \beta_{j} + \xi^{c}_{jits} \quad \text{and} \quad v^{c}_{ijts} = X^{c}_{ijt} \alpha_{i} + \varepsilon^{c}_{ijts}$$

where $s$ denotes a single draw. In order to simulate each market, I utilize the matching with contracts algorithm from Hatfield and Milgrom (2005), and adapt it to fit my setting. I leave a more detailed discussion of the matching algorithm for Appendix C.\textsuperscript{19} I present results from the counterfactual simulations in Table 8.

<table>
<thead>
<tr>
<th>Table 8: Counterfactual Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Total Matches</td>
</tr>
<tr>
<td>Fraction Syndicated</td>
</tr>
<tr>
<td>Total Expected IPOs</td>
</tr>
<tr>
<td>IPO Probability</td>
</tr>
<tr>
<td>Relative to Baseline</td>
</tr>
<tr>
<td>Total Company Utility</td>
</tr>
<tr>
<td>Total VC Utility</td>
</tr>
<tr>
<td>Total Utility</td>
</tr>
</tbody>
</table>

Notes: Results are computed based on 1,000 error draws. The fraction syndicated is the number of syndicated matches relative to the number of total simulated matches. The total expected IPOs is the sum of IPO probability across all simulated market observations. Total utility measures are the sum of all simulated utilities across all markets.

It is important to note that for each tax rate I consider, the demand for venture capital is assumed to be the same (i.e. the number of companies is fixed at 2,665). Consequently, it is possible to have an excess supply of venture capital, as seen when the tax rate is 10%, because VCs have enough capacity to fund 3,822 companies. Alternatively, when the tax rate is 20% and 35% there is a shortage of venture capital, because VC capacity is not large enough for all companies to receive investments. The impact of an excess and a shortage of venture capital supply will become clear when examining the distribution of simulated utilities. Interestingly, the increase in the tax rate does not result in an increase in the fraction of matches that are syndicated. One potential explanation is that the VCs more likely to syndicate an investment due to restrictive capital constraints exit the market. This leaves the VCs with more available capital who would prefer not to syndicate. Additionally,\textsuperscript{19} I provide a more in depth explanation of the simulation procedure in the online appendix.
the probability of an IPO is relatively constant across different tax rates, which seems counter-intuitive given the drop in syndication. However, the VCs and companies that match under higher tax rates are those of higher quality, and, thus, more likely to achieve an IPO. These two forces act in opposition, which results in the stable IPO probability. The utility measures in the lower half of Table 8, compare the total utility across all simulations for companies, VCs, and their sum, relative to the baseline. Here we see that increasing the tax rate clearly decreases total utility, with $\tau^{cg} = 20\%$ being the closest to the baseline.\(^{20}\)

**Company Utility.** Although the statistics are informative, the utility distributions under each tax rate offer the most insight. Figure 5 plots the distribution of two different measures of company utility for each tax rate and the baseline. Panels A and B correspond to the mean utility for each company with the expectation taken over all simulated markets. Panels C and D plot the distributions for the mean utility with the expectation taken over simulated markets in which a given company was matched. These are denoted by $\bar{u}_i$ and $\tilde{u}_i$ respectively for company $i$, and are defined below:

$$
\bar{u}_i = \frac{1}{S} \sum_{s=1}^{S} u_{\mu^s(i)s} \quad \text{and} \quad \tilde{u}_i = \frac{1}{|S(i)|} \sum_{s \in S(i)} u_{\mu^s(i)s}
$$

(27)

where $S(i)$ is the set of simulated markets for which company $i$ matches with a VC. The distinction between these two utility measures is important. The first captures the overall effect from a change in the tax rate. The second highlights the effect on expected utility for the companies that do not get left out of the market. One would expect that the first measure will be smaller under a higher tax rate, but this is not necessarily the case for the second measure. It is possible that the average utility is quite high, because the remaining VCs with whom the companies can match are of the highest quality. In Panel A, the welfare decreasing impact of the tax rate can be seen, with the distribution of expected utilities showing companies to be worse off under a capital gains tax rate of 35%. Panel B shows that company expected utility distributions dominate those with higher tax rates in the First Order Stochastic (FOSD) sense. In Panel C, the shift in the distributions is apparent, however, it is not the case that the utilities for the companies matched under the highest tax rate are higher than under the other tax rates. The answer as to why lies in the contract choice. Since the fraction of matches that are syndicated is lowest when the tax rate is highest, a portion of the companies switch from syndicated to non-syndicated contracts as the tax rate increases. This acts as a downward force on the distribution of utilities. Another interesting feature of Panel C is the change in dispersion. The smaller set of VCs in the market causes the variance of utilities under the highest tax rate to be smaller than the variance for the both the baseline and the tax rate of 20%. This decrease in dispersion is reflected in the CDFs in Panel D. While the lowest tax rate still FOSD the other tax rates, the two intermediate cases do not dominate the highest tax rate in FOSD or SOSD sense.

**VC Utility.** In the case of the VC utility distributions, I consider two slightly different measures than those defined in (27). Since VCs can match with more than one company, the difference between the mean utility a VC receives from its portfolio and the total utility provides useful information about the impact of tax rate changes on VC welfare. These two measures are given below, respectively:

$$
\bar{v}_j = \frac{1}{|S(j)|} \sum_{s \in S(j)} \left( \sum_{i \in \mu^s(j)} \frac{v_{i\mu^s(i)s}}{|\mu^s(j)|} \right) \quad \text{and} \quad \tilde{v}_j = \frac{1}{|S(j)|} \sum_{s \in S(j)} \sum_{i \in \mu^s(j)} v_{i\mu^s(i)s}
$$

(28)

\(^{20}\)This is expected given the tax rate during the sample period is between 15% and 16.05%.
Figure 5: Distribution of Company Expected Utilities. Notes: (i) Panel A plots the distribution of expected company utilities under each scenario. (ii) Panel B plots the corresponding CDFs for the distributions in Panel A. (iii) Panel C plots the distribution of weighted expected company utilities under each scenario. (iv) Panel D plots the corresponding CDFs for the distributions in Panel C. (v) The vertical lines in Panels A and C represent the mean of each distribution. The mean for the baseline distribution is in black.
where $S(j)$ is the set of simulations for which VC $j$ matches with at least one company. The first measure captures the average quality of the companies with whom a VC matches, which does not necessarily decrease as a VC’s portfolio shrinks. On the other hand, the second measure accounts for the total payoff the VC receives, which is more sensitive to tax changes than the mean. The corresponding VC utility distributions are plotted in Figure 6.

Figure 6: Distribution of VC Expected Utilities. Notes: (i) Panel A plots the distribution of VC mean portfolio utility under each scenario. (ii) Panel B plots the corresponding CDFs for the distributions in Panel A. (iii) Panel C plots the distribution of expected total VC utility under each scenario. (iv) Panel D plots the corresponding CDFs for the distributions in Panel C. (v) The vertical lines in Panels A and C represent the means of each distribution.

The distributions in Panel A are the most intriguing. Here we see that the average utility that a VC receives from each company in its portfolio is actually higher when the tax rate is higher. There are two reasons this pattern emerges. First, all VCs reduce capacity,
which leaves them with only their most preferred companies. Second, the reduction in capacity for the best VCs implies that some of the companies with whom they were previously matched become available. As a result, lower quality VCs can invest in these higher quality companies. Consequently, the ordering of the distributions is reversed relative to the company ordering. The distribution of utilities under the highest tax rate FOSD the distributions with lower tax rates. Of course, the higher per company utility a VC receives under the highest tax rate does not imply that it is better off under this policy. Clearly the expected ordering of distributions is restored when examining the total utility measure in Panels C and D. However, the differences in distributions across tax rates is not as large as it is for companies. While the best companies are far worse off under the highest tax rate, the best VCs do not experience such a large decrease in total utility. For these VCs, the positive effect on total utility from increasing mean portfolio quality mitigates the negative effect from capacity reduction.

**Contract Switching.** Although company and VC exits impact the utility distribution, it is also affected by companies and VCs choosing different contracts under different tax rates. Due to the complicated interactions in the matching market, the impact of the tax rate on contract choice is not immediately clear. I examine the change in contract choice by calculating the fraction of simulated markets in which a company accepts a given contract or exits the market. Mathematically this is given by:

\[
\text{Contract Share}_{ik} = \frac{1}{S} \sum_s \mathbf{1}\{c^s(i) = k\}
\]

where \(c^s(i)\) is the equilibrium contract for company \(i\) in simulation \(s\), and \(k \in \{\text{syndicated, non-syndicated, exit}\}\). This measure can be viewed as the simulated probability that a given contract is chosen a company. I also calculate the share of a VC’s portfolio that is comprised of a given contract. The syndicated portfolio share is given by:

\[
\text{Portfolio Share}_{jk} = \frac{1}{S} \sum_s \sum_{i \in \mu^s(j)} \frac{1\{c^s(i) = k\}}{|\mu^s(j)|}
\]

This measure is the expected portfolio share of a given contract. In Panels A through C of Figure 7, the distributions of company contract shares are plotted for the three capital gains tax rates. The distribution of company exits in Panel A is excluded, because all of the companies are always matched due to the excess supply of venture capital. Under the higher tax rates, we see that always receiving an investment becomes less and less common, with the probability of a company remaining unmatched increasing. The change in share of syndicated and non-syndicated contracts as the tax rate increases is particularly striking. When the tax rate is 10%, the excess capacity of VCs gives the companies more bargaining power in the matching market. Consequently, they are more likely to match in their more preferred contract. As the tax rate increases, there are two forces that reduce the syndicated contract share. First, company exits cause the distributions for both contracts to shift to the left. Second, when companies do not exit, the likelihood that they choose a non-syndicated contract increases. For the highest tax rate, the smaller VC capacity implies some companies will remain unmatched. The threat of not matching severely reduces their bargaining power, and one concession they make to remain in the market is to accept non-syndicated contracts. This contract switching by companies prevents the utility distribution under the highest tax rate from FOSD the distributions under the lower tax rate, as mentioned when discussing Figure 5.

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21The difference in the right tail of the utility distributions for VCs and companies illustrates this fact.
Figure 7: Distribution of Contract Choice. Notes: (i) The contract share for companies is the share of simulated markets in which they select a given contract. Since no companies go unmatched under the lowest tax rate, the distribution is excluded from Panel A. (ii) Panels A through C plot the contract share distributions for different capital gains tax rates. (iii) The portfolio share is the share of each contract in a VC’s portfolio for each simulation. The distribution of exits in Panels E and F are excluded, because VCs either never exit or always exit. (iv) Panels D through F plot the portfolio share distributions for different capital gains tax rates. (v) The vertical lines in each panel represent the means of each distributions.

The portfolio share distributions are plotted in Panels D through Panels F in Figure 7. Since changes in the tax rate directly affect VC exits, VCs with a predicted portfolio size less than or equal to zero are always excluded from the market. However, it is still possible for VCs to be left out due to the sorting mechanism. When the tax rate is lowest, the excess supply of venture capital leads to a non-degenerate distribution of VC exits. However, when some VCs are excluded from the market due to tax increases, the fixed demand for venture capital results in the VCs with positive predicted portfolio size always matching with at least one company. This leads to a mass point at zero, so the distribution of exits is excluded from Panels E and F. Interestingly, syndicated contracts comprise a larger share of
a VC’s portfolio for higher tax rates. This result illustrates the impact of capital constraints on VC contract choice. There are two potential forces driving this pattern. First, VCs switch to syndicated contracts for companies in whom they invest for all tax rates. Second, as their portfolio sizes shrink, VCs are more likely to keep companies with whom they previously chose syndicated contracts. It should be noted that the difference between the contract distributions is smaller under the highest tax rate than the intermediate one. This suggests an interplay between bargaining power and capital constraints. As more VCs exit the market, company bargaining power decreases. As a result, VCs are more likely to match under the contract they prefer. This acts in opposition to the impact of capital constraints, which leads to the slightly nonlinear relationship between tax rate and portfolio share. In examining the contract and portfolio share together, we can glean information about which VCs are choosing which contracts. Even though the distributions of syndicated and non-syndicated contracts are similar under the highest tax rate for companies, there is still a discrepancy in the portfolio share. The implication is that VCs with more capacity are more likely to invest alone, while VCs with less capacity are more likely to syndicate their investments. This result neatly illustrates the finance-related motive for syndication through the effect of capital constraints on portfolio size.

In terms of guidance for policy-makers, the counterfactual exercise shows the complicated effect a tax policy can have on the VC industry due to the matching between VCs and companies. As always, the policy decision largely rests on the desired outcome. Whether it is maximizing tax revenue, increasing the number of IPOs, or improving the probability of a company achieving an IPO; the optimal tax rate is not the same. However, the exercise has demonstrated the benefit of using a two-sided matching model as a tool for policy analysis.

8. Conclusion

This paper develops a two-sided matching model with contracts in order to control for the impact of endogenous matching between VCs and companies on outcome estimates of the effect syndication has on company IPO probability. Since integrating the likelihood function is computationally intractable, I obtain Bayesian using a Gibbs sampler and data augmentation. I use a sample of 2,665 first round investments in companies by VCs from 2004 to 2012. I show that estimates of the VA from syndication are robust to sorting between VCs and companies. Additionally, I demonstrate that the finance-related, selection, and value-added motivations for syndication act are all important factors in a VC’s syndication decision. These motivations are interrelated and do not necessarily operate on the same direction, which makes accounting for them especially important. Furthermore, I find that sorting along the unobservables changes over time, which leads to the standard Probit model both overestimating and underestimating the VA from syndication at different years in the sample. Finally, I conduct counterfactual simulations in which I examine the impact that changes in the long-term capital gains tax rate have on the matching market. I show that VC and company exit, along with contract switching, have varying effects on both the number of expected IPOs as well as the distributions of company and VC utilities.

Although this paper takes an important step forward, there remains a great deal to be examined in both the VC industry and the econometric methods for dealing with endogenous matching. One potential extension in VC industry would be obtaining more granular data regarding contract terms in order to determine how these are affected by sorting, and the impact on company outcomes. Another avenue for future research that could be particularly fruitful is adapting a matching model to account for the switching of lead investors over the lifetime of a company. There is a substantial amount of information available
between the initial investment round and the company outcome. A two-sided matching model that accounts for rematching in each investment round would capture the changing preferences for match partners that impact company outcomes. However, the econometric method for estimating a dynamic matching model has not yet been developed, so this must be accomplished first. Therefore, there are multiple opportunities for future research.
A. Inequalities Characterizing Equilibrium Matching

The unique equilibrium matching is characterized by a set of inequalities for utilities on each side of the market that utilize the equilibrium solution concept from Section 3.3 which is based upon the results in Hatfield and Milgrom (2005). These inequalities are derived similarly to Chen (2013), but the potential for different matchings for a given VC-company pair due to contract choice requires more detailed bounds on the utilities. Since the bounds are derived in the same way for any given market, I exclude the $t$ subscript for exposition.

**Company Utility Bounds.** First, consider the bounds for company utility, which are given by $u^c_{ji}$ and $\overline{u}^c_{ji} \forall c \in C$.\footnote{These bounds are VC-company-contract specific, however, in the estimated model I use VC-contract specific bounds for company utility. This simply requires one further step for the lower and upper bounds to derive the maximum and minimum, respectively, for a VC-contract.} Furthermore, let $c^*(i)$ and $c'(i)$ represent the equilibrium and off-equilibrium path contracts for company $i$. Since there are only two contracts to choose from $c'(i)$ is necessarily a singleton. If company $i$ and VC $j$ are not matched, then the no blocking condition dictates that in the event the VC prefers $(i,j,c)$ to its least preferred match; it must be that the company does not prefer the deviation to its equilibrium match. Thus, the first step in characterizing the upper bound on the utility from the pair is:

$$u^c_{ji} = \begin{cases} 
    u^c_{c^*(i)} & \text{if } v^c_{ij} > \min_{i' \in \mu(j)} v^c_{i'j} \text{ and } i \notin \mu(j) \\
    \infty & \text{otherwise} \end{cases} \tag{A1}$$

The next step deals with the potential for the first contract offered by the VC being rejected. Let the set of VCs not matched with company $i$, but would prefer to do so under a contract $c$ as $f(i)$. Additionally, let $r(j)$ be the set of companies matched with VC $j$ for whom the VC prefers the off-equilibrium contract (i.e. the first contract offer made by the VC was rejected). Define an alternative upper bound for a given company, $i$, as:

$$\tilde{u}^c_i = \begin{cases} 
    \max_{(j',c) \in f(i)} \left\{ 0, \max_{(j',c) \in f(i)} u^c_{j'i} \right\} & \text{if } v^c_{ij} < v^c_{ij} \text{ and } i \in \mu(j) \\
    \infty & \text{otherwise} \end{cases} \tag{A2}$$

When the first offer made by VC $j$ is rejected, it must be that the company held a better offer from a different VC. Consequently, it must be that the utility the company receives from this other VC must be higher than the off-equilibrium contract offer made by VC $j$. Since the company utility in the model is VC-contract specific, the alternative upper bound on each contract for VC $j$ is:

$$\tilde{u}^c_j = \min_{(i,c(i)) \in r(j)} \tilde{u}^c_i \tag{A3}$$

The utility in (A3) must be low enough to ensure that all companies matched with VC $j$ have a better outside option than the off-equilibrium contract if they rejected the first offer from VC $j$. Finally, the upper bounds on the utility under each contract for VC $j$ are

$$\overline{u}^c_j = \min_{i \in I \setminus \overline{u}^c_j, \tilde{u}^c_j}.$$  

Next I derive the lower bounds for company utility. The lower bound for a given VC-company pair is:

$$u^c_{ji} = \begin{cases} 
    \max_{(j',c) \in f(i)} u^c_{j'i} & \text{if } i \in \mu(j) \text{ and } c = c^*(i) \\
    0 & \text{otherwise} \end{cases} \tag{A4}$$

The lower bound for each contract for each contract for a VC is $\underline{u}^c_j = \max_{i \in I} \underline{u}^c_{ji}$. 

---
**VC Utility Bounds.** Now consider the VC utility bounds given by $v^c_{ij}$ and $\pi^c_{ij}$, $\forall c \in C$. Unlike the company utility bounds, these are VC-company-contract specific. The first step in characterizing the upper bounds on VC utility is:

$$v^c_{ij} = \begin{cases} \min_{i' \in \mu(j)} v^{c(i')}_{i'j} & \text{if } u^c_j > u^{c(i')}_{\mu(i)} \text{ and } j \neq \mu(i) \\ \infty & \text{Otherwise} \end{cases} \quad (A5)$$

Again, the next step is to deal with the fact that the first offer may be rejected, however, it is slightly more involved for VC utilities. These intermediate upper bounds are given by:

$$\tilde{v}^{c(i)}_{ij} = \begin{cases} v^{c(i)}_{ij} & \text{if } u^{c(i)}_j > u^{c(i)}_{ij}, (i, c'(i)) \in r(j), \text{ and } j = \mu(i) \\ \infty & \text{Otherwise} \end{cases} \quad (A6)$$

$$\tilde{v}^{c'}_{ij} = \begin{cases} v^{c'(i)}_{ij} & \text{if } (i, c'(i)) \notin r(j) \text{ and } j = \mu(i) \\ \infty & \text{Otherwise} \end{cases} \quad (A7)$$

The upper bound for the on-equilibrium contract utility, in (A6), is equal to the off-equilibrium contract utility in the event the VC offered it first and the company rejected it. The upper bound for the off-equilibrium contract utility, in (A7), is equal to the equilibrium contract utility if the first contract offer was accepted. The final upper bounds for each contract are $\pi^c_{ij} = \min \{ \tilde{v}^{c(i)}_{ij}, \tilde{v}^{c'}_{ij} \}$.

Now I derive the lower bounds for the VC utilities. Note that for the VC utilities, it is possible to derive lower bounds for both the on-and off-equilibrium contracts. Let $f(j)$ denote the set of companies that are not matched with VC $j$ in equilibrium, but would prefer to do so under a given contract $c$. The lower bound for the equilibrium contracts is:

$$\underline{v}^{c(i)}_{ij} = \begin{cases} \max \left\{ 0, \max_{(i', c') \in f(j)} \bar{v}^{c(i)}_{i'j} \right\} & \text{if } u^{c(i)}_j > u^{c(i)}_{ij}, u^c_j > u^{c(i')}_{\mu(i')}, (i, c'(i)) \in r(j), \text{ and } j = \mu(i) \\ \max \left\{ \bar{v}^{c(i)}_{ij}, \max_{(i', c') \in f(j)} \bar{v}^{c(i')}_{i'j} \right\} & \text{if } u^c_j > u^c_{\mu(i')}, (i, c'(i)) \notin r(j), \text{ and } j = \mu(i) \\ 0 & \text{Otherwise} \end{cases} \quad (A8)$$

The lower bound for off-equilibrium contracts is:

$$\underline{v}^{c'(i)}_{ij} = \begin{cases} \max \left\{ \bar{v}^{c(i)}_{ij}, \max_{(i', c') \in f(j)} \bar{v}^{c'(i')}_{i'j} \right\} & \text{if } u^{c(i)}_j > u^{c(i)}_{ij}, u^c_j > u^{c(i')}_{\mu(i')}, (i, c'(i)) \notin r(j), \text{ and } j = \mu(i) \\ 0 & \text{Otherwise} \end{cases} \quad (A9)$$

The bounds derived above permits the following characterization of a unique equilibrium matching, $\mu^*$:

$$\mu = \mu^* \iff u^c_j \in \left( \underline{v}^c_{ij}, \pi^c_{ij} \right) \text{ and } v^c_{ij} \in \left( \tilde{v}^{c(i)}_{ij}, \pi^c_{ij} \right) \forall (i, j, c) \in M \quad (A10)$$

**B. Conditional Posterior Distributions**

Throughout this section the distributions are derived by completing the square. Let a random vector have the density given below:

$$\pi(\theta) = C_0 \times \phi \left( \theta^t M_0 \theta + 2\theta^t N_\theta + C_1 \right) \quad (A11)$$

The resulting distribution of $\theta$ is $N \left( M_0^{-1} N_\theta, M_0^{-1} \right)$. I use this notation through the section.
**B.1. Condition Distribution of Outcome**

The indicator function $1\{y_{ij} = 1\}$ equals one when a company achieves an IPO and zero otherwise. The conditional density is given by:

$$
\pi(Y^*, \mu, U, V|Y^*_{ij}, \Omega, \theta) = C \times 1\{y_{ij} = 1\} \times \phi(Y^*_{ij} - \mathbf{W}_{ij}^c \Gamma - \delta(v^c_{ij} - \mathbf{X}_{ij}^c \alpha) - \kappa(u^c_{ji} - Z^c_{ji} \beta))
$$

(A12)

Given the conditional density, the mean is:

$$
\mu_{Y^*} = \mathbf{W}_{ij}^c \Gamma + \delta(v^c_{ij} - \mathbf{X}_{ij}^c \alpha) + \kappa(u^c_{ji} - Z^c_{ji} \beta)
$$

(A13)

and the variance is equal to one. Since the outcome is a Probit model, this follows a normal distribution given by $N(\mu_{Y^*}, 1)$ s.t. $\mu_{Y^*} \in (-\infty, 0)$ if $y_{ij} = 0$ and $\mu_{Y^*} \in [0, \infty)$ if $y_{ij} = 1$.

**B.2. Conditional Distribution of Utilities**

The distribution of the utility from a given contract $c$ depends on whether a given VC $j$ matches with a given company $i$ under that contract. I use the notation, $(i, c) \in \mu(j)$, to denote the company-contract pair that are matched with VC $j$ in equilibrium. For a given VC utility, $v_{ij}^c$. The terms $M_{v_{ij}}$ and $N_{v_{ij}}$ are:

$$
M_{v_{ij}} = 1 + 1\{(i, c) \in \mu(j)\} \delta^2
$$

(A14)

$$
N_{v_{ij}} = \mathbf{X}_{ij}^c \alpha + 1\{(i, c) \in \mu(j)\} \delta (Y^*_{ij} - \mathbf{W}_{ij}^c \Gamma + \delta \mathbf{X}_{ij}^c \alpha - \kappa(u^c_{ji} - Z^c_{ji} \beta))
$$

(A15)

For a given company utility, $u_{ij}^c$. The terms $M_{u_{ij}}$ and $N_{u_{ij}}$ are:

$$
M_{u_{ij}} = 1 + \sum_{(i, c) \in \mu(j)} \kappa^2
$$

(A16)

$$
N_{u_{ij}} = Z^c_{ji} \beta + \kappa \sum_{(i, c) \in \mu(j)} (Y^*_{ij} - \mathbf{W}_{ij}^c \Gamma - \delta(v^c_{ij} - \mathbf{X}_{ij}^c \alpha) + \kappa Z^c_{ji} \beta)
$$

(A17)

**B.3. Conditional Distribution of Parameters**

The parameters in the model are constant across markets, so now the terms must be summed across all markets in the model. The distribution of the outcome equation parameters contained in $\Gamma$ is given by the terms $M_{\Gamma}$ and $N_{\Gamma}$ which are:

$$
M_{\Gamma} = \Sigma_{\Gamma}^{-1} + \sum_{t=1}^T \sum_{(i, j, c) \in \mu_t} \mathbf{W}_{ij}^c \mathbf{W}_{ij}^c
$$

(A18)

$$
N_{\Gamma} = -\Sigma_{\Gamma}^{-1} \Gamma - \sum_{t=1}^T \sum_{(i, j, c) \in \mu_t} \mathbf{W}_{ij}^c (Y^*_{ij} - \delta(v^c_{ij} - \mathbf{X}_{ij}^c \alpha) - \kappa(u^c_{ji} - Z^c_{ji} \beta))
$$

(A19)

the terms $M_{\alpha}$ and $N_{\alpha}$ which are:

$$
M_{\alpha} = \Sigma_{\alpha}^{-1} + \sum_{t=1}^T \left[ \sum_{(i, j, c) \in M_t} \mathbf{X}_{ij}^c \mathbf{X}_{ij}^c + \sum_{(i, j, c) \in \mu_t} \delta^2 \mathbf{X}_{ij}^c \mathbf{X}_{ij}^c \right]
$$

(A20)

$$
N_{\alpha} = -\Sigma_{\alpha}^{-1} \alpha - \sum_{t=1}^T \left[ \sum_{(i, j, c) \in M_t} \mathbf{X}_{ij}^c v^c_{ij} - \sum_{(i, j, c) \in \mu_t} \mathbf{X}_{ij}^c \delta (Y^*_{ij} \mathbf{W}_{ij}^c \Gamma - \delta u^c_{ij} - \kappa(u^c_{ji} - Z^c_{ji} \beta)) \right]
$$

(A21)
the terms $M_\delta$ and $N_\delta$ which are:

$$M_\delta = \Sigma^{-1}_\delta + \sum_{t=1}^{T} \left[ \sum_{(i,j,c)\in M_t} (v_{ij}^c - X_{ij}^c \alpha)(v_{ij}^c - X_{ij}^c \alpha)' \right]$$

$$N_\delta = - \Sigma^{-1}_\delta \beta - \sum_{t=1}^{T} \left[ \sum_{(i,j,c)\in M_t} (v_{ij}^c - X_{ij}^c \alpha)(v_{ij}^c - X_{ij}^c \alpha)' \right]$$

where $\mu^c(j)$ denotes the set of companies matched to VC $j$ under contract $c$.

the terms $M_\kappa$ and $N_\kappa$ which are:

$$M_\kappa = \Sigma^{-1}_\kappa + \sum_{t=1}^{T} \left[ \sum_{(i,j,c)\in M_t} |\mu^c(j)|(w_{ji}^c - Z_{ji}^c \beta)(w_{ji}^c - Z_{ji}^c \beta)' \right]$$

$$N_\kappa = - \Sigma^{-1}_\kappa \kappa - \sum_{t=1}^{T} \left[ \sum_{(i,j,c)\in M_t} (w_{ji}^c - Z_{ji}^c \beta)(w_{ji}^c - Z_{ji}^c \beta)' \right]$$

C. Contract Matching Algorithm

In this section I discuss the matching algorithm used in the estimation procedure. First I introduce some notation to provide a more detailed description of the matching procedure. Then I explain the matching algorithm when VCs offer contracts.

Let $C_t(I)$ and $C_t(J)$ be the set of offered contracts and the set of unrejected contracts through iteration $t$ respectively. Additionally, let $C$ be the total set of contracts, and $R_I$ and $R_J$ be the set of contracts not offered and the cumulative set of rejected offers respectively. Finally, $q_j$ is the quota for VC $j$. Below I describe the company-offering algorithm. The VC-offering algorithm is largely the same, except that VCs offer their $q_j$ most preferred contracts versus companies offering their single most preferred contract. The general definition for a stable allocation of contracts adapted from Hatfield and Milgrom (2005) states that if $(\mathcal{C}_I, \mathcal{C}_J) \in \mathcal{C}^2$ is a solution to the system of equations:

$$\mathcal{C}_I = C - R_J(\mathcal{C}_J) \quad \text{and} \quad \mathcal{C}_J = C - R_I(\mathcal{C}_I)$$

then $\mathcal{C}_J \cap \mathcal{C}_I$ is a stable set of contracts, and $\mathcal{C}_J \cap \mathcal{C}_I = \mathcal{C}_I(\mathcal{C}_I) = \mathcal{C}_J(\mathcal{C}_J)$.

VC-Offering Algorithm. For the VC-offering algorithm, the chosen set of contracts for VCs and companies is $\mathcal{C}_I(C)$ and $\mathcal{C}_J(C)$ respectively. The algorithm to find the stable set of contracts when VCs offer contracts proceeds as follows:
$t = 0$: The algorithm is initialized with $C_I(0) = C$ and $C_J(0) = \emptyset$.

$t = 1$: VCs offer their $q_j$ most preferred contracts from $C$ to VCs. Companies hold their most preferred contract and reject others. Then the set of offered contracts is updated: $C_J(1) = C - R_J(C_I(1))$.

$t = \tau$: At iteration $\tau$, VCs offer their most preferred $q_j$ contracts from $C_I(\tau) = C - R_J(C_J(\tau - 1))$ to companies. The companies hold their most preferred contracts and reject others. Then the set of offered contracts is updated: $C_J(\tau) = C - R_J(C_I(\tau))$.

$t = \tau + 1$: Suppose the fixed point is reached after $\tau + 1$ iterations, so that: $C_J(\tau) = C_J(\tau + 1)$ and $C_I(\tau) = C_I(\tau + 1)$. Thus, the set of stable contracts is $C_J(\tau) \cap C_I(\tau)$.

References


Roth, Alvin E. and Marilda Sotomayor (1990), “Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis.”


