CGPA WORKING PAPER
2026-17

Trade Fragmentation, International Cartels, and Welfare: How Does Domestic Market Structure Matter?

Delina E. Agnosteva
Constantinos Syropoulos
Yoto V. Yotov

DREXEL UNIVERSITY
Center for
Global Policy Analysis
LeBow College of Business

Trade Fragmentation, International Cartels, and Welfare: How Does Domestic Market Structure Matter?[†]

Delina E. Agnosteva
Pennsylvania State University

Constantinos Syropoulos[‡]
Drexel University

Yoto V. Yotov
Drexel University

Current Version: July 31, 2026

Abstract: We characterize collusive pricing in an international cartel spanning two host countries and pooling incentive constraints across markets, under general demand restricted only by a mild curvature condition that admits constant elasticity. Under domestic monopoly, fragmentation can weaken collusion and raise host welfare, whereas richer profit opportunities abroad strengthen collusion and may lower it; hosts prefer moderate barriers to either free trade or complete separation. Under domestic competition both results reverse. Whether fragmentation disciplines cartels thus depends on market structure in their home countries. Since collusion requires no trade between members, prices rather than trade flows carry the identifying information.

JEL Classification: D43, F10, F12, F13, F15, L12, L13, L41

Keywords: Fragmentation, oligopoly, multimarket interactions, cartel discipline, collusive pricing, trade costs, domestic market structure.

[†]Contact Information: Delina Agnosteva – dagnosteva@psu.edu; Constantinos Syropoulos – c.syropoulos@drexel.edu; Yoto Yotov – yotov@drexel.edu.

[‡]Corresponding author.

1 Introduction

An international cartel operating across multiple countries does not police each market in isolation. Because a deviation anywhere triggers punishment everywhere, members pool a single incentive constraint across markets, allowing slack in one market to sustain collusion in another (Bernheim and Whinston, 1990). Trade costs enter this pooled constraint twice with opposing signs: they shrink the prize from invading a partner's market—relaxing the constraint—while potentially sheltering a firm at home if the agreement breaks down, softening the disciplinary threat and tightening the constraint. Which channel dominates depends on whether reversion to competition leaves firms with trade-cost-sensitive profits at home, which is governed by host-country market structure. This article shows that this mechanism flips both the positive and normative economics of trade barriers.

The question is timely because trade costs are rising. After decades of declining trade costs and deepening market integration, the world economy has entered a period of *economic fragmentation*. Tariff wars, sanctions, export controls, and the reorganization of supply chains along geopolitical lines have raised the costs of cross-border commerce and are widely expected to keep doing so (Fajgelbaum et al., 2020; Aiyar et al., 2023). The retreat now reaches even the deepest integration arrangements: on July 1, 2026, following repeated threats by President Trump to abandon the accord, the United States declined to renew the United States–Mexico–Canada Agreement (USMCA) at its first scheduled joint review, placing North American free trade on a path of annual renegotiations with termination on the table.[1] The welfare consequences of such fragmentation are routinely assessed under the presumption of competitive conduct—efficiency losses tallied market by market. Yet many traded industries are dominated by large firms (Head and Spencer, 2017; Neary, 2003), and a non-trivial number are cartelized across the very borders along which trade barriers are now rising. This raises the article's central question: can economic fragmentation discipline collusion among internationally active firms and thereby improve welfare, or does it instead entrench the very cartels it is presumed to weaken? We show that the answer is conditional, hinging on an observable feature of the cartel's home markets. Moreover, because collusion is sustained without trade among cartel members, fragmentation affects collusive prices and welfare while leaving no trace in the trade flows that competition authorities typically monitor.

That such collusion persists is not in doubt: multimarket cartels remain prevalent despite decades of strengthened antitrust enforcement. Since 1990, the European Commission alone has prosecuted over 160 international cartels, imposing fines in excess of €32 billion on the

[1] The agreement remains in force pending the annual reviews and expires in 2036 absent renewal; U.S. tariffs on Canadian and Mexican steel, aluminum, and automobiles were imposed alongside the review. See USTR, "Ambassador Greer Issues Statement on the USMCA Joint Review," July 1, 2026.

firms involved.[2] Many of these cartels involved homogeneous goods—such as sugar, cement, gasoline, steel, bitumen, and various vitamins—and operated across national borders, often affecting markets as broad as the entire European Economic Area (Connor, 2010).[3] A common strategy among these cartels has been to fix prices above competitive levels and to allocate market shares according to agreed-upon rules (Harrington, 1991, 2006; Bos and Harrington, 2010). Territorial allocation is the dominant such rule: roughly eighty percent of the international cartels detected by European and American authorities in recent decades allocated territories or specific customers to their members (Levenstein and Suslow, 2011). The geography of our model mirrors these arrangements. In the choline chloride cartel, for instance, North American and European producers agreed to withdraw from each other's home markets while dividing third-country markets in Latin America and Asia (Connor, 2010; European Commission, 2005). The cartel's North American bloc was itself a cross-border coalition—Bioproducts and DuCoa of the United States and the Chinook Group of Canada—so that intra-cartel trade costs of the kind we study ran through the very region whose integration the USMCA now leaves in doubt. Because such cartels operate across markets separated by trade costs—and because their members routinely export to third countries as well—the trade cost is a policy-relevant determinant of cartel discipline, and the margin through which fragmentation acts.

To address our question, we develop a model in which each of two countries ("cartel hosts") is home to $n \geq 1$ firms that produce a homogeneous good at a common constant marginal cost, form a price-setting cartel, and may export to each other's market at per-unit cost t and to a third destination—the rest of the world (ROW)—at per-unit cost τ.[4] Collusion is sustained in an infinitely repeated game with grim trigger strategies that threaten reversion to the Bertrand-Nash equilibrium of the static price game, and firms pool their incentive constraints across all markets (Bernheim and Whinston, 1990). A distinguishing feature of our framework is that demand functions are general, restricted only by the requirement that they not be excessively convex—our Assumption (A2) below. Within this framework we ask four questions, all phrased in the language of fragmentation. *First*, which collusive price agreements are self-enforcing, and how does this set vary with trade costs? *Second*, how does the stability of any targeted agreement depend on trade costs

[2] Statistics on contemporary international cartels are available at: `https://competition-policy.ec.europa.eu/antitrust-and-cartels/cartels-cases-and-statistics_en`.

[3] As per the European Commission, in the case of vitamins, there are indications that the collusive practices were felt worldwide. See COMP/E-1/37.512 Vitamins: `https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32003D0002&from=EN`.

[4] Alternatively, the two "hosts" may be interpreted as two geographically segmented regional markets of a single country, with t an internal trade friction. Under this interpretation, our analysis asks whether internal fragmentation—and the cartel's participation in export markets—affects domestic collusion and welfare. As we show, the cartel's presence in the world market as an exporter can indeed be welfare-reducing for its host.

and on the profitability of exporting to *ROW*? *Third*, which agreements does an optimizing cartel select—the "efficient" agreements—when its incentive compatibility constraint (ICC) is active? *Fourth*, how do fragmentation and changes in *ROW* profitability affect the welfare of cartel hosts and of the world?

Our main answer is that the consequences of fragmentation hinge on market structure *within* the cartel hosts. Under domestic monopoly ($n = 1$), the punishment phase involves limit pricing: reversion to Bertrand-Nash competition leaves each firm a profit that *rises* with trade costs, because higher trade costs shelter it from foreign entry.[5] Fragmentation therefore erodes the severity of punishment, and this channel can destabilize collusion causing the efficient collusive price to be non-monotone in trade costs and host welfare to be hump-shaped: intra-regional trade cost increases are welfare-enhancing when the initial level of trade costs is moderate but their complete elimination is welfare-reducing. Access to profitable exports in *ROW* reinforces the cartel by relaxing its ICC, and—strikingly—improvements in *ROW* profit opportunities can immiserize cartel hosts by strengthening collusion at home—a possibility we establish for general demand (Proposition 5) and characterize in closed form under linear demand (Proposition 6). Under domestic competition ($n \geq 2$), by contrast, Bertrand reversion yields zero profits *regardless* of trade costs: the punishment channel vanishes, and only the deviation channel operates. Every conclusion then gets reversed. Fragmentation is unambiguously pro-collusive: the minimum discount factor capable of sustaining any agreement falls with trade costs, collusive prices weakly rise with them, and sustaining the monopoly price never requires more patience than the classic threshold for $2n$ Bertrand competitors. Access to *ROW* now *undermines* the cartel, to the point that an impatient cartel optimally restrains its own profits in *ROW*—moderating its extraction abroad to protect collusion at home. Host welfare may still rise with mild fragmentation, but only through beggar-thy-neighbor rent extraction from third markets; global welfare falls monotonically as trade barriers rise. The answer to our central question is thus conditional—and the condition identifies precisely how domestic market structure matters: rising trade barriers discipline international cartels only when each host market is monopolized domestically.

This article contributes to the literature examining the impact of economic integration on multimarket collusion and welfare.[6] Pinto (1986) builds upon Brander and Krugman's

[5] The punishment convention is therefore substantive: under maximal punishments in the sense of Abreu (1986, 1988), punishment payoffs are zero regardless of market structure and this channel is shut down. We discuss the role of the convention—and why it is the source, rather than a fragility, of our reversal—in Section 2.

[6] A series of articles focuses on the effects of trade policy on cartels comprised of domestic and foreign firms operating in a single market. Davidson (1984) analyzes the effect of lower tariffs on collusion between domestic and foreign firms competing in quantities. Syropoulos (1992) explores the effects of tariffs and quotas on the behavior of a differentiated-goods, quantity-setting cartel and the welfare implications of

(1983) model by allowing repeated interactions and shows that, for specific values of the discount factor, trade will be prohibited if firms choose the monopoly output. Lommerud and Sørgard (2001) offer a homogeneous-good duopoly model where collusion is sustained through trigger strategies and examine the effects of trade costs on the sustainability of collusion. Schröder (2007) incorporates *ad valorem* tariffs and fixed trade costs and demonstrates that reductions in trade costs can lower cartel stability. Bond and Syropoulos (2008) develop a homogeneous-good segmented-markets duopoly model in which quantity-setting firms pool their incentive constraints; they show that cartel agreements may support the emergence of intra-industry trade and that economic integration may enhance collusive stability and reduce welfare. Akinbosoye et al. (2012) study price competition with differentiated goods and find that, if goods are close substitutes and trade costs are high, trade liberalization facilitates collusion. Ashournia et al. (2013) employ a segmented-markets differentiated-goods Cournot duopoly and show that reductions in trade costs enhance collusion when the initial level of trade costs is sufficiently low. Bhattacharjea and Sinha (2015) analyze price-setting cartels that sustain territorial allocation at monopoly prices and show that the effect of trade costs on the sustainability of that arrangement reverses with the number of firms per territory—and, relatedly, that exogenous third-market profitability relaxes the cartel's incentive constraint under Nash reversion but tightens it under maximal punishments. Agnosteva et al. (2024) allow quantity-setting cartel members to export to a rest-of-the-world aggregate and find that, when the ICC binds, all markets become strategically linked, so that regional economic integration need not improve welfare in all markets. Deltas et al. (2012) adopt a Hotelling model of horizontal differentiation and find that collusion can raise both national welfare and consumer surplus relative to price competition.

Relative to this literature, our contribution is fourfold. First, we provide, to our knowledge, the first complete analysis of a price-setting international cartel with third-market exports under general demand, characterizing the full set of efficient agreements rather than only the stability of maximal collusion.[7] This focus equips the framework for welfare analysis: because stability analyses characterize whether a targeted agreement—typically maximal collusion—can be sustained, rather than which agreement an optimizing cartel would select when its incentive constraints bind, they are not designed to address welfare questions. Within this literature the only welfare analyses we are aware of are the quantity-setting ones of Syropoulos (1992), Bond and Syropoulos (2008), and Agnosteva et al. (2024). Ours is the first for self-enforcing collusion in prices, and it delivers welfare

trade liberalization.

[7]Harrington (1991) also studies collusive prices below the monopoly level, but in a heterogeneous-cost duopoly serving a single market. In our setting, the relevant heterogeneity arises endogenously from trade costs.

statements for cartel hosts, for ROW, and for the world.

Second, the generality of demand here is disciplined by a single restriction on a primitive of the model: Assumption (A2), which limits the degree of convexity of demand. The standard practice in this literature (e.g., Harrington, 1991; Bhattacharjea and Sinha, 2015) is to assume that each firm's profit function is strictly concave in price. This restriction excludes the constant-elasticity demand function—a workhorse of the trade literature—for which profits inevitably turn convex at sufficiently high prices.[8] Because (A2) restricts demand free of cost parameters, one directly verifiable condition disciplines profit functions in every market simultaneously—domestic, partner, and ROW—and is portable to other multimarket pricing problems. The gain is not merely one of coverage. The article's most striking welfare result—that improvements in profit opportunities in ROW can immiserize cartel hosts—holds under *any* admissible demand (Proposition 5), and the very convexity that a maintained profit-concavity assumption would exclude is what upgrades it from a local statement to a global one (Theorem B1 in Appendix B), with constant-elasticity demand as the leading case. Several of the objects we characterize also admit closed forms; most notably, under domestic competition in the absence of exports to ROW, the minimum discount factor is independent of the shape of demand.

Third, we place the market structure of the cartel hosts (domestic monopoly versus domestic competition) at the center of the analysis. Bhattacharjea and Sinha (2015), our closest antecedent, observe, for cartels that allocate territories and sustain monopoly prices, that the number of firms per territory reverses the effect of trade costs on the sustainability of that particular arrangement. We show that the reversal is far more general and far more consequential, and we depart from their analysis in three respects. (i) The reversal applies to the entire set of efficient agreements at every discount factor, and thus to the collusive price itself, an object that their territorial arrangement holds fixed. (ii) It extends to the cartel's endogenous choices in ROW, where access to third markets switches from reinforcing collusion under domestic monopoly to undermining it under domestic competition, to the point that an impatient cartel finds it optimal to restrain its own profits there (Proposition 7(c)); in their framework a third market enters only through its exogenous profitability, so this margin does not arise. (iii) It reverses the welfare economics of fragmentation, severing the alignment between host and global welfare; in their setting, as in most of the stability literature, welfare is not analyzed. Thus, where they establish a reversal in the stability of a single price agreement, we establish a reversal of the positive and the normative economics of fragmentation alike, indexed by an observable feature of the cartel hosts' home markets.

Fourth, we show that the transmission of trade costs to cartel discipline operates through

[8] With $D(p) = Ap^{-\varepsilon}$, profit $\pi(p, c)$ is strictly concave in price if and only if $p < (\varepsilon + 1)c/(\varepsilon - 1)$, so global profit concavity fails. By contrast, (A2) holds for every $\varepsilon > 1$.

fundamentally different channels under price and quantity competition, with an implication for inference that we regard as important in its own right. In contrast to the quantity-setting analyses of Bond and Syropoulos (2008) and Agnosteva et al. (2024), collusive agreements here never involve costly cross-hauling (two-way trade in the homogeneous product between the cartel hosts). Trade between the hosts is therefore absent under collusive and competitive conduct alike, so fragmentation affects collusion and welfare without leaving a trace in trade flows. This feature carries uncomfortable implications for regulators who infer market integration from trade data, and equally for empirical work that draws inferences about collusive conduct from trade flows: trade data are silent on both integration and collusion, and price behavior, not trade volumes, carries the identifying information—the collusive price responds non-monotonically to trade costs under domestic monopoly and monotonically under domestic competition. By implication, price-based screens, rather than trade-based ones, are the appropriate detection instruments in this setting (Section 4, Remark 3).

The rest of the article is organized as follows. Section 2 presents the model. Sections 3 and 4 study the benchmark of domestic monopoly ($n = 1$): Section 3 solves the cartel's constrained optimization problem, analyzes the stability of collusion and its dependence on trade costs and ROW profits, and characterizes efficient price agreements; Section 4 examines the welfare implications of fragmentation and of enhanced profitability in ROW. Section 5 turns to domestic competition ($n \geq 2$) and establishes the reversals described above. Section 6 concludes. All proofs not given in the text appear in the appendices; Appendix C discusses the punishment convention and shows that our two analyses bracket the space of credible punishments.

2 The Model

We consider a homogeneous-good oligopoly in which each of two countries—referred to as "cartel hosts"—is home to a set of $n \geq 1$ firms. These firms, which form a price-setting cartel, share access to a constant returns to scale technology characterized by a common marginal cost of production c (≥ 0). In addition to serving their domestic markets, cartel members can export to each other's market as well as to ROW.

The per-unit cost for a firm to supply its own domestic market is normalized to zero. In contrast, shipping a unit of output to the other cartel host incurs a cost of $t \geq 0$, while exporting to ROW involves a per-unit cost of $\tau \geq 0$. These trade costs may reflect geographic frictions, administrative barriers, or trade taxes. For future reference, we denote by c_{ij} the per-unit cost incurred by firm $i \in I \equiv \{1, 2\}$ to produce and deliver its product to market $j \in J \equiv I \cup \{ROW\}$.

The demand function $D_j(p) > 0$ in market j is assumed to be twice differentiable and decreasing in price $p \in [c_{ij}, \tilde{p}_j]$, where $\tilde{p}_j > \max_i\{c_{ij}\}$ s.t. $D_j(p) = 0$ iff $p \geq \tilde{p}_j$.[9] For tractability, we also adopt the following assumptions:

(A1) $D_i(p) = D(p)$ for $i \in I$.

(A2) $\eta(p) \equiv 2 - \gamma(p) > 0$, where $\gamma(p) \equiv \frac{D(p)D''(p)}{D'(p)^2}$.

Assumption (A1) abstracts from differences in demand functions among cartel hosts. As we will see, this assumption—together with our assumption of symmetry on trade costs—simplifies the cartel's optimization problem and places at center stage the dependence of collusive prices on trade costs. However, $D_{ROW}(\cdot)$ and $D(\cdot)$ need not be identical. Assumption (A2) constrains the curvature of demand $D(p)$. Clearly, (A2) is satisfied when $D(p)$ is concave in price because it implies $\gamma(p) \leq 0$ and thus $\eta(p) > 0$. Still, (A2) admits the possibility of convex demand functions, including the one with constant price elasticity.[10] The key feature of (A2) is that it limits the degree of convexity in demand.

Two additional comments on (A2) merit emphasis here. First, as shown in Lemma A1 in Appendix A, (A2) implies that a firm's profit function in each market is strictly quasi-concave in price. This guarantees the existence of a unique maximum. It also ensures that the profit function is strictly concave for all relevant prices below the one associated with the monopoly level. Unlike an assumption of profit concavity, (A2) is imposed on a *primitive* of the model: it can be verified directly from a demand estimate or functional form, and the single condition then disciplines profit functions in every market simultaneously—domestic, partner, and *ROW*—even though delivered costs differ across these markets. This matters because the ICC aggregates profit *differences* across markets with different costs, so market-by-market concavity assumptions would obscure the one primitive restriction doing the work. Assumption (A2) is invoked exactly where the analysis requires it—the monotonicity of the minimum discount factor in the collusive price (Lemma 3) and the stability analysis of Proposition 1—and, notably, is *not* needed for the corresponding monotonicity under domestic competition on the export-deviation region (Lemma 4($b.i$)), a contrast that is itself informative about where demand curvature bites. Second, (A2) is less restrictive than Marshall's "Second Law of Demand" (MSLD), which requires the absolute value of the elasticity of demand to rise with price.[11]

[9] An alternative formulation is to require $\lim_{p\to\infty} D(p) = 0$, as in the case of constant elasticity of demand $D(p) = Ap^{-\varepsilon}$.

[10] It is straightforward to verify that, in the constant elasticity of demand case, $D(p) = Ap^{-\varepsilon}$, we have $\eta = \frac{\varepsilon-1}{\varepsilon}$, which is clearly positive when $\varepsilon > 1$.

[11] This condition has been proven analytically useful and empirically relevant in imperfectly competitive settings, including monopolistic competition with heterogeneous firms (Melitz, 2018). For clarity and intuition, define $\varepsilon(p) \equiv -pD'(p)/D(p)$ and note that MSLD is tantamount to requiring $\theta(p) \equiv 1+\frac{1}{\varepsilon(p)} - \gamma(p) > 0$. One may now rewrite $\eta(p)$ in (A2) as $\eta(p) = 1 - \frac{1}{\varepsilon(p)} + \theta(p)$. It follows that MSLD implies $\eta(p) > 0$ and

Throughout, we assume that the demand function in ROW satisfies (A2) as well, so that Lemma A1 applies market by market—a fact we use repeatedly, most prominently in Section 5.

Let us now pay closer attention to profit functions. Denote with $\pi_{ij}(p, c_{ij}) = (p - c_{ij})D_j(p)$ the operating profit function of firm i in market $j \in J$, let superscript "m" stand for "**m**onopoly", and define $p^m_{ij} \equiv \arg\max_p \pi(p, c_{ij})$ firm i's monopoly price in market j. Our assumptions of symmetric trade costs and identical demand functions among cartel hosts generate a *mirror-image* property of prices and profit functions across markets. They also give rise to the following ranking of monopoly prices and profits: $p^m \equiv p^m_{ii} < p^m_{ij}$ and $\pi^m \equiv \pi^m_{ii} > \pi^m_{ij}$ for $i \neq j \in I$. Corresponding relations arise for monopoly prices and profits in ROW. Specifically, $p^m_{ROW} \equiv p^m_{iROW}$ and $\pi^m_{ROW} \equiv \pi^m_{iROW}$ for $i \in I$.[12]

To ensure that entry by firm i into market j ($\neq i$) through exporting is meaningful (i.e., it generates a non-negative profit $\pi_{ij} \geq 0$), we assume that trade costs t and τ satisfy the following inequalities:

$$\text{(A3)} \qquad c_{ij} = c + t \leq p^m \qquad \text{and} \qquad c_{iROW} = c + \tau \leq p^m_{ROW} \qquad \text{for } i \neq j \in I.$$

A direct implication of Assumption (A3) is that there exist prohibitive trade cost levels $\bar{t} \equiv p^m - c$ and $\bar{\tau} \equiv p^m_{ROW} - c$ that imply exporting to a cartel host and to ROW is unprofitable for trade cost levels $t \geq \bar{t}$ and $\tau \geq \bar{\tau}$, respectively.[13] Assumption (A3), which can be rewritten as $t \in [0, \bar{t}\,]$ and $\tau \in [0, \bar{\tau}]$, essentially requires all firms to be strategically active in all markets (at $t = \bar{t}$ and $\tau = \bar{\tau}$, only weakly so).

As is customary in the literature on price competition among firms that supply a homogeneous good, the firm that sets the lowest price in a market captures the entire demand there. In contrast, if a subset of firms announce the same price, any division of demand between them—including the symmetric division—is admissible. Importantly for our purposes here, Assumptions (A1)-(A3) render it natural for us to focus on mirror-image (resp., symmetric) divisions of demand and profits in markets 1 and 2 (resp., ROW). Our goal in this context is to identify and characterize the most profitable subgame perfect equilibrium of prices that firms can sustain in an infinitely repeated price-setting supergame subject to their multimarket incentive compatibility constraint (ICC). As we will see shortly, this

thus (A2)—provided, of course, $\varepsilon(p) > 1$. The converse does not hold: (A2) admits demand functions that exhibit larger degrees of convexity than MSLD permits. As such, (A2) is less restrictive than MSLD.

[12] It is useful to keep in mind that $p^m = p^m(c)$ while $p^m_{ij} = p^m_{ij}(c+t)$ for $i \neq j \in I$ and $p^m_{ROW} = p^m_{ROW}(c+\tau)$. More generally, since $\partial^2 \pi_{ij}/(\partial p \partial c_{ij}) = -D'_j(p) > 0$, the first-order necessary condition (FOC) for profit maximization together with the strict quasi-concavity of π_{ij} in p imply $dp^m_{ij}/dc_{ij} > 0$. This ensures that $p^m_{1j} \gtreqless p^m_{2j}$ and $\pi^m_{1j} \lesseqgtr \pi^m_{2j}$ as $c_{1j} \gtreqless c_{2j}$ for any $j \in J$.

[13] As noted earlier, $p^m_{ROW}(c + \tau)$ is an increasing function of τ. Therefore, $\bar{\tau}$ is the solution to $\tau = p^m_{ROW}(c+\tau) - c$ (which is also unique). In contrast, the definition of $\bar{t}$ is simpler because $p^m(c)$ is independent of t.

constraint will ensure that the representative firm's global payoff under a collusive pricing agreement does not fall short of the weighted sum of the global payoff it would secure under an optimal deviation from that agreement and its per-period global payoff in the "punishment" phase, which we identify with infinite reversion to the "Bertrand-Nash" equilibrium of the single-period price game.

Henceforth, we identify the "Bertrand-**N**ash" equilibrium of the one-shot price game with superscript "n", a "collusive price **a**greement" with superscript "a", and an optimal "**d**eviation" from it with superscript "d". Throughout, prices set in every market are publicly observed at the end of each period, so a deviation in any one market triggers, from the following period onward, the punishment in all of them. We can now make the following observations.

Bertrand-Nash equilibrium: Starting with cartel hosts, note that the presence of trade costs $t \in (0, \bar{t}]$ gives a pricing advantage to local firms. Importantly, in this case, the number of firms n residing in cartel hosts plays a crucial role in equilibrium pricing. We must distinguish between the following two possibilities.

(i) If $n = 1$, it is potential exporters that pose a competitive threat to local firms. In this case, a local firm i will be able to charge a price marginally below the marginal cost $c + t$ of any potential exporter j $(\neq i \in I)$ to its market thereby discouraging entry. This limit pricing behavior implies $p^n = c + t$ in every cartel host and a global per-period profit $\pi^n \equiv \pi(p^n, c) = (p^n - c)\, D\,(p^n) = tD\,(c + t)$, which is increasing in $t \in (0, \bar{t}]$.[14]

(ii) On the other hand, if $n > 1$, the competitive threat comes from domestic rivals (and not from foreign exporters) because no local firm has a cost advantage over any other firm in that market. In this case, competition induces firms to behave as perfect competitors thus yielding $p^n = c$ and $\pi^n = 0$.

Consider now pricing in ROW. Because all firms face identical delivery costs $c + \tau$ to that market, price competition will induce them to undercut each other's price; therefore, $p^n_{ROW} = c + \tau$ and $\pi^n_{ROW} = 0$.

Remark 1 *(Bertrand punishment and market structure) The representative firm's global profit in the Bertrand-Nash equilibrium can be described as follows: (i) if $n = 1$, then $\pi^n = tD(c + t)$, so punishment involves limit pricing whose severity* diminishes *as trade costs rise; (ii) if $n > 1$, then $\pi^n = 0$,* regardless *of trade costs.*

This distinction is fundamental: Sections 3 and 4 study the benchmark of domestic monopoly ($n = 1$); Section 5 shows that outcomes change profoundly under domestic competition ($n \geq 2$).

[14]This profit is solely due to domestic operations and $\bar{t} \equiv p^m - c$.

Because the distinction operates entirely through the punishment phase, the punishment convention itself merits comment at the outset. We follow the bulk of the literature on trade costs and collusion (e.g., Lommerud and Sørgard, 2001; Bond and Syropoulos, 2008; Akinbosoye et al., 2012) in adopting grim-trigger reversion to the Bertrand-Nash equilibrium: the harshest punishment that is subgame perfect without requiring firms to coordinate on non-Nash play off the equilibrium path. This is a substantive choice, and it is the source rather than a fragility of the reversal we document. Under maximal punishments in the sense of Abreu (1986, 1988), punishment payoffs would be driven to the minmax value of zero *regardless* of domestic market structure, and the limit-pricing profits at the heart of our benchmark would vanish. What determines the sign of the fragmentation effect, then, is whether punishment-phase profits depend on trade costs: under Nash reversion they do so only when each host market is monopolized, whereas under maximal punishments they never do. Our two analyses moreover *bracket* the space of punishment conventions, so that the model with optimal penal codes—whatever the harshest credible punishment turns out to be—is nested between two cases this article fully characterizes. Appendix C develops the bracketing argument and the credibility and empirical considerations that favor Nash reversion.

Collusion: Under a collusive price agreement, it is profitable for firms to separate the markets of cartel hosts geographically, charge a uniform price $p \in [p^n, p^m]$ in each of these markets, and allocate demand among local firms equally. Formally, in this case, the representative firm will enjoy a per-period profit of $\pi^a \equiv \frac{1}{n}\pi(p,c) = \frac{1}{n}(p-c)D(p)$ in its domestic market.[15] (Hereafter, for convenience and to avoid notational clutter, we will occasionally suppress superscript "a"from collusive agreements.)

Owing to symmetry, firms will also have an incentive to set a common price $p_{ROW} \in [p^n_{ROW}, p^m_{ROW}]$ in ROW and allocate $\frac{1}{2n}$ of the market demand there to each (exporting) firm. This generates a per firm profit $\frac{1}{2n}\pi^a_{ROW}$, where $\pi^a_{ROW} \equiv (p_{ROW} - c - \tau)\, D_{ROW}\,(p_{ROW})$. For obvious reasons, π^a (resp., π^a_{ROW}) is increasing in p (resp., p_{ROW}), and attains a maximum at p^m (resp., p^m_{ROW}). Moreover, π^a_{ROW} and π^m_{ROW} are decreasing in $\tau \in [0, \bar{\tau}]$.

Optimal deviations: Here we describe the benchmark case $n = 1$; as Section 5 shows, the deviation payoff takes the same form under domestic competition. A firm i that deviates from a collusive agreement will: (i) continue setting the collusive price p in its own market

[15]Note that $t < \bar{t}$ implies $p^n < p^m$ and $\pi^n < \pi^m$. That is, cartel members can obtain higher profits under collusion than under price competition because collusion enables them to separate respective markets geographically. This is so because "withdrawal"(Bernheim and Whinston, 1990) of inefficient suppliers (i.e., potential exporters) saves on trade costs. In the limiting case that $t = \bar{t}$ we have $p^n = p^m$ and $\pi^n = \pi^m$. Also note that, while unequal allocations of demand are also feasible, equal divisions are natural in the symmetric setting we are considering.

i to capture $\pi^a = \pi(p, c)$.[16] (ii) undercut the collusive price p in its partner's market j ($\neq i$), by charging a price p^d marginally below p and thereby capturing the entire demand in that market to secure a profit $\pi^d \equiv \pi(p^d, c+t) = \left(p^d - c - t\right) D\left(p^d\right)$; (iii) undercut the collusive price p_{ROW} in ROW, by setting a price p^d_{ROW} marginally below p_{ROW}, to capture the entire market demand there and enjoy profit $\pi^d_{ROW} \equiv \pi_{ROW}(p^d_{ROW}, c+\tau) = \left(p^d_{ROW} - c - \tau\right) D_{ROW}\left(p^d_{ROW}\right)$. Since the difference between the deviating price p^d and the collusive price p (resp., p^d_{ROW} and p_{ROW}) is infinitesimal, we will treat these prices interchangeably when there is no potential for confusion. For the same reason, we will also set $\pi^d \approx \pi(p, c+t)$ and $\pi^d_{ROW} \approx \pi_{ROW}(p_{ROW}, c+\tau)$.

We can illustrate the above ideas with two prominent functional forms for demand: (i) the linear case, where $D(p) = \alpha - \beta p$ for α and $\beta > 0$; and (ii) the case of constant elasticity, $D(p) = Ap^{-\varepsilon}$ for $A > 0$ and $\varepsilon > 1$. In the first case, we find $\bar{t} = \frac{1}{2}(\alpha/\beta - c)$, $p^m = \frac{1}{2}(\alpha/\beta + c)$, and $\pi^m = \frac{1}{4}\beta(\alpha/\beta - c)^2$. In the second case, we find $\bar{t} = c/(\varepsilon - 1)$, $p^m = \frac{\varepsilon}{\varepsilon-1}c$ and $\pi^m = Ac^{-(\varepsilon-1)}(\varepsilon-1)^{\varepsilon-1}\varepsilon^{-\varepsilon}$. Since the cost of delivering a unit of a good to ROW is $c + \tau$, the corresponding prices and profits in ROW can be obtained by replacing c with $c + \tau$ in these expressions.

Two additional points ought to be kept in mind in the analysis to follow. First, $t < \bar{t}$ implies $p^n < p^m$ while $t = \bar{t}$ implies $p^n = p^m$.[17] Second, for any price $p \in [p^n, p^m]$ that might be targeted by the cartel as a potential agreement, define $t_x \equiv p - c$. Then, if the actual intra-regional trade cost t is not smaller than t_x, a deviating firm's profit would satisfy $\pi^d = 0$.

The above observations enable us to write the representative firm's per-period **global** profit Π^k ($k \in \{a, d, n\}$) in the domestic-monopoly benchmark ($n = 1$) as a function of collusive prices $\mathbf{p} \equiv (p, p_{ROW})$ and trade costs (t, τ) as follows:

$$\Pi^a(\mathbf{p}, \tau) = \pi(p, c) + \frac{1}{2}\pi_{ROW}(p_{ROW}, c+\tau) \tag{1a}$$

$$\Pi^d(\mathbf{p}, t, \tau) \approx \pi(p, c) + \pi(p, c+t) + \pi_{ROW}(p_{ROW}, c+\tau) \tag{1b}$$

$$\Pi^n(t) = \pi(p^n, c), \tag{1c}$$

where, once again, $p^n = c + t$. The first and second terms in the right-hand side (RHS) of Π^a in (1a) identify the representative cartel member's per-period collusive profit in its own market and in ROW, respectively.[18] The three terms in the RHS of Π^d in (1b) respectively capture a representative cartel member's per-period profits under an optimal deviation from

[16] The deviating firm has no incentive to adjust the collusive price in its own market because under the collusive agreement its rival has already agreed to withdraw from this market.

[17] These points can be established by noting that $p^n = c + t$ and $\bar{t} = p^m - c$.

[18] Recall that cartel members earn no profit under a collusive agreement in their partner's markets.

a collusive agreement in: (i) the member's own market; (ii) its cartel partner's market; and (iii) ROW's market. The RHS of (1c) is self-explanatory. The properties of per-period profits described earlier imply that both Π^a and Π^d are concave in $\mathbf{p}$. Moreover, Π^a is decreasing in τ while Π^d is decreasing in t and τ. (Note that Π^a does not depend on t.) In contrast, Π^n is increasing in $t \in [0, \bar{t})$ (because $p^n = c + t$).

As discussed in the next section, the global profits defined above will determine the cartel's ICC. To pave the way for that analysis two remarks in connection with global profits Π^a and Π^d deserve some emphasis. First, in contrast to the corresponding global profits in quantity-setting games, Π^a and Π^d always arise here in the absence of cross-hauling under a collusive agreement because there is no advantage to the cartel of maintaining a presence in a member's market via exports.[19] Second, a cartel member's per-period profit in ROW under an optimal deviation is (approximately) twice as large as its per-period profit under a collusive price agreement there. (Under domestic competition the corresponding factor is $2n$; see Section 5.)

3 Self-Enforcing Price Agreements under Domestic Monopoly

Throughout this section and the next we focus on the benchmark of domestic monopoly ($n = 1$), so that the Bertrand-Nash punishment involves limit pricing at $p^n = c+t$; domestic competition ($n \geq 2$) is taken up in Section 5. We are now in a position to examine the representative cartel member's optimization problem. For presentation purposes, we proceed as follows. First, in an effort to understand the cartel's problem, we study the key traits of its ICC and the roles played by the types of markets within which cartel members interact, the trade costs they face in these markets, and their time preferences. Second, we explore the stability/sustainability of feasible cartel agreements. In the third subsection we solve the cartel's problem and characterize (constrained) efficient price agreements.

3.1 Incentive Compatibility Constraint

Denote with $\delta \in (0, 1)$ the firms' common discount factor. The representative cartel member will not wish to deviate from a collusive price agreement $\mathbf{p} \equiv (p, p_{ROW})$ if its global profit under the agreement Π^a is not lower than the weighted sum of its global profit under an

[19] Bond and Syropoulos (2008) and Agnosteva et al. (2024) show that an optimizing cartel may find it appealing to engage in costly cross-hauling when firms interact in quantities because exporting enhances the sustainability of agreements by reducing deviation payoffs. This feature is absent here because, when a cartel member deviates from an agreement, its pricing allows it to capture the entire market demand. As a consequence, its payoff is independent of the quota (i.e., export volume) that might be allocated to it or its partner under a collusive agreement.

optimal deviation Π^d from that agreement and the Bertrand-Nash payoff Π^n. That is,

$$\Phi(\mathbf{p}, t, \tau, \delta) \equiv \Pi^a(\mathbf{p}, \tau) - (1-\delta)\Pi^d(\mathbf{p}, t, \tau) - \delta\Pi^n(t) \geq 0. \tag{2}$$

Let $\mathcal{F}(t, \tau, \delta) \equiv \{\mathbf{p} \mid \Phi(\mathbf{p}, t, \tau, \delta) \geq 0 \text{ and } \mathbf{p}^n \leq \mathbf{p} \leq \mathbf{p}^m\}$ be the set of incentive compatible price agreements. The cartel's optimization problem can then be described as: $\max_{\mathbf{p}} \Pi^a(\mathbf{p}, \tau)$, s.t. $\mathbf{p} \in \mathcal{F}(t, \tau, \delta)$. Henceforth, we will identify the solution to this problem with a "*" and we will refer to it as the "efficient" agreement. Clearly, our ability to characterize this solution hinges on our understanding of the nature of $\Phi(\cdot)$ in (2), so we start there.

Substituting the global profit functions in (1) into (2) allows us to rewrite $\Phi(\cdot)$, after some rearrangement and simplification, as

$$\Phi(p, p_{ROW}, t, \tau, \delta) = -\frac{1}{2}\Psi(p, t) + \left(\delta - \frac{1}{2}\right)\Omega(p, p_{ROW}, t, \tau) \tag{3}$$

where

$$\Psi(p, t) \equiv \pi(p, c+t) - \pi(p, c) + \pi(p^n, c) \tag{4a}$$

$$\Omega(p, p_{ROW}, t, \tau) \equiv \pi(p, c+t) + \pi(p, c) - \pi(p^n, c) + \pi_{ROW}(p_{ROW}, c+\tau) \tag{4b}$$

It is worth recording at the outset that Ω is exactly the per-period profit differential between deviation and punishment, $\Omega \equiv \Pi^d - \Pi^n$. This identity is what allows the analysis of domestic competition in Section 5—where $\Pi^n = 0$, which causes Ω to collapse to the deviation payoff Π^d—to be conducted with the same objects. Lemmas 1 and 2 below prepare the ground for our characterization of $\Phi(\cdot)$.

Lemma 1 *(Features of Ψ and Ω in the ICC) Suppose $\mathbf{p} \in [\mathbf{p}^n, \mathbf{p}^m]$. Then:*

a) $\Psi(p^n, t) = 0$ *and* $\Omega(p^n, p_{ROW}, \cdot) = \pi_{ROW}(p_{ROW}, c+\tau)$.

b) $\Psi(p, t) \geq 0$ *for* $p > p^n$ *and* $t \geq 0$, *with equality for* $t = 0$.

c) $\Phi_\delta = \Omega(\cdot) \geq 0$, *with equality for* $\mathbf{p} = \mathbf{p}^n$.

Part (a) of Lemma 1 follows readily by: (i) noting that $p = p^n$ leads to $\pi(p, c+t) = 0$ (which implies that the profit gain to firm i that considers defecting in market $j \neq i \in I$ is zero), and $\pi(p, c) = \pi(p^n, c)$; and by applying these observations to the definitions of the $\Psi(\cdot)$ and $\Omega(\cdot)$ functions in (4). Part (b) can be proven by utilizing the definitions of the per-period profits in the definition of $\Psi(\cdot)$ and by simplifying the resulting expression to rewrite it as

$$\Psi(p, t) \equiv t\,[D(p^n) - D(p)] \geq 0.$$

The sign of $\Psi(\cdot)$ above follows from our focus on $p \in (p^n, p^m]$ and the facts that $D'(p) < 0$ and $t \geq 0$. Part (c) can be established by differentiating Φ in (3) with respect to δ and noting that: (i) $\Omega(\cdot) > 0$ for $\mathbf{p} \neq \mathbf{p}^n$, because $\pi(p,c) > \pi(p^n, c)$ for the trade cost levels considered; and (ii) $\Omega(p^n, p^n_{ROW}, \cdot) = 0$, since $\pi_{ROW}(p^n_{ROW}, c+\tau) = 0$.

The next lemma highlights the importance of the value of the discount factor in the determination of the nature of the ICC. As such, this lemma lays the foundation for our analysis of "efficient" price agreements.

Lemma 2 *(The ICC and the minimum discount factor) For* $\mathbf{p} \in (\mathbf{p}^n, \mathbf{p}^m]$*, define* $\underline{\delta}(p,\cdot) \equiv \frac{\Pi^d - \Pi^a}{\Pi^d - \Pi^n} = \frac{1}{2} + \frac{\Psi(p,t)}{2\,\Omega(p, p_{ROW}, t, \tau)}$*. Then:*

a) $\Phi = (\delta - \underline{\delta})\,\Omega$*. Hence the ICC in (2) holds if and only if* $\delta \geq \underline{\delta}(p,\cdot)$*, and* $\mathcal{F}(t,\tau,\delta)$ *consists of* $\mathbf{p}^n$ *together with the lower contour set of* $\underline{\delta}$ *at height* δ*.*

b) If $\mathbf{p} = \mathbf{p}^n$*, then* $\Phi(\mathbf{p}^n, t, \tau, \delta) = 0$ *for all* $\delta \in [0,1)$*, so* $\mathcal{F}(t,\tau,\delta)$ *is never empty.*

c) $\underline{\delta} \geq \frac{1}{2}$*, with equality if and only if* $\Psi(p,t) = 0$*. Consequently, if* $\delta \leq \frac{1}{2}$ *and* $t > 0$*, no price* $\mathbf{p} \in (\mathbf{p}^n, \mathbf{p}^m]$ *satisfies the ICC. Conversely, if* $\delta \in (\frac{1}{2}, 1)$*, a nonempty subset of such prices satisfies the ICC whenever* $\pi^*_{ROW} > 0$ *or, if* $\pi^*_{ROW} = 0$*, whenever* t *is sufficiently small;*[20] *the complete characterization follows in Lemma* 3 *and Proposition* 1*.*

The fact that $\Phi(\mathbf{p}^n, \cdot) = 0$ for all $\delta \in [0,1)$ implies that $\mathcal{F}(t,\tau,\delta)$ is not empty. Therefore, unsurprisingly, the ICC is always satisfied when the cartel adopts the Bertrand-Nash prices $\mathbf{p}^n$. Part (a), which follows from (3) and the definitions in (4) upon division by $\Omega > 0$, organizes everything that follows: characterizing self-enforcing—and, ultimately, efficient—agreements amounts to characterizing the function $\underline{\delta}$. We refer to $\underline{\delta}$ interchangeably as the *patience requirement* of an agreement: the least patient a cartel can be and still sustain it. The same identity, with $\Pi^n = 0$, underlies the analysis of domestic competition in Section 5 (Lemma 4(a)), so the two market structures receive a unified treatment. Parts (b) and (c) follow from part (a) and Lemma 1, and explain how the discount factor δ and trade costs t matter for prices other than $\mathbf{p}^n$.

Part (c) reveals that the ICC cannot be satisfied for any of the prices considered if $\delta \leq \frac{1}{2}$ and $t > 0$. This is so because, for $\mathbf{p} \in (\mathbf{p}^n, \mathbf{p}^m]$, $\Psi(\cdot) > 0$ by Lemma 1(b) and $\Omega(\cdot) > 0$, so that $\underline{\delta} = \frac{1}{2} + \Psi/(2\Omega)$ strictly exceeds $\frac{1}{2}$. Interestingly, the ICC is satisfied if $\delta = \frac{1}{2}$ and there are no trade costs to cross-hauling ($t = 0$), because $\Psi(\cdot) = 0$ in this case. The second statement in part (c) is of central importance to our aims here because it identifies a key condition on the value of the discount factor (specifically, $\delta \in (\frac{1}{2}, 1)$) under which the ICC can be satisfied for prices above $\mathbf{p}^n$—unconditionally so when the cartel earns profits in

[20] Part (c) anticipates a reduction established later in this subsection: for $\delta > \frac{1}{2}$ the cartel sets $p^*_{ROW} = p^m_{ROW}$, so that $\pi^*_{ROW} = \pi^m_{ROW}$. Until that point π^*_{ROW} may simply be read as the collusive profit $\pi_{ROW}(p_{ROW}, c+\tau)$ implied by the agreed ROW price.

ROW, and for sufficiently small trade costs otherwise. Nonetheless, an important problem remains: We still do not know how the set of incentive-compatible prices $\mathcal{F}(t, \tau, \delta)$ depends on its arguments. This is the issue we plan to address next.

As a segue to the above task, we start with an important implication of $\delta > \frac{1}{2}$: $\Phi(\cdot)$ is increasing in π_{ROW} in this case.[21] In words, the higher the cartel profit in ROW, the larger the slack in the ICC, and thus the higher the potential for the cartel to benefit from multimarket contact by using that slack to support more collusive outcomes in all markets. But $\arg\max_p \pi_{ROW} = p^m_{ROW}$; therefore, $p^*_{ROW} = p^m_{ROW}$ and $\pi^*_{ROW} = \pi^m_{ROW}$ for $\delta > \frac{1}{2}$. Recognition of this fact simplifies the cartel's problem by reducing the dimensionality of its ICC and by enabling us to study it solely by focusing on the dependence of $\Phi(\cdot)$ on $p \in [p^n, p^m]$.[22] Nonetheless, as we will see shortly, the solution to this problem and the characterization of that solution are non-trivial.

Consider now the dependence of $\Phi(\cdot)$ on its arguments. Total differentiation of $\Phi(\cdot)$ gives

$$d\Phi = \Phi_\delta d\delta + \Phi_{\pi^*_{ROW}} d\pi^*_{ROW} + \Phi_p dp + \Phi_t dt.$$

We have already shown that $\Phi_\delta = \Omega > 0$ for $p > p^n$ (Lemma 1 (c)), and that $\Phi_{\pi^*_{ROW}} > 0$ for $\delta > \frac{1}{2}$. With these ideas in mind, then, let us consider the signs of Φ_p and Φ_t. From the definition of $\Phi(\cdot)$ in (3) and its component parts in (4) it is straightforward for one to show that

$$\Phi_p = -(1-\delta)\,\pi_p(p, c+t) + \delta\pi_p(p, c) \tag{5a}$$

$$\Phi_t = -(1-\delta)\,\pi_t(p, c+t) - \delta\pi_p(p^n, c). \tag{5b}$$

Inspection of Φ_p in (5a) reveals a potential ambiguity in its sign. This is so for the following reasons. An increase in p undermines the ICC by raising the profitability of a cartel member's deviation in its partner's market (captured by $\pi(p, c+t)$ in the RHS of (5a)). At the same time, however, the increase in p improves its collusive profit in a cartel member's own market (captured by $\pi(p, c)$ in the RHS of (5a)) for $t > 0$. Moreover, we have $\pi_p(p, c+t) > \pi_p(p, c) \geq 0$.[23] The problem is that the weights $1-\delta$ and δ attached to these marginal profits satisfy $1-\delta < \delta$ because we are focusing on $\delta \in \left(\frac{1}{2}, 1\right)$. In other words,

[21] This is due to the fact that Φ is increasing in Ω which, in turn, is increasing (linearly) in π_{ROW}.

[22] Recognition of the fact that $\pi^*_{ROW} = \pi^m_{ROW}$ enables us to focus directly on π^*_{ROW} itself rather than indirectly through its dependence on trade costs τ and demand conditions in ROW. A notable advantage of this approach is that it allows us to treat π^*_{ROW} as a parameter whose variation helps unveil the importance of ROW.

[23] This is so because $\pi_p(p, c+t) = \pi_p(p, c) - tD'(p) > 0$ for $p \in (p^n, p^m]$, whereas $\pi_p(p, c) \geq 0$ for the same prices (with equality for $p = p^m$), and $t > 0$ and $D'(p) < 0$.

the sign of Φ_p appears to be ambiguous because it is unclear which of the above conflicting effects dominates. In what follows we show that the shape of the demand function $D(p)$ resolves this ambiguity—though not by signing Φ_p globally, which is not possible: by Lemma 2(a), $\Phi_p = -\underline{\delta}_p \Omega + (\delta - \underline{\delta})\, \Omega_p$, so the sign of Φ_p is pinned down only where the ICC binds. What the shape of demand delivers is the sign of $\underline{\delta}_p$, and that is what the cartel's problem requires: with $\underline{\delta}_p > 0$, the set of incentive-compatible prices is an interval with p^n at its lower end, so the constrained optimum is its upper endpoint.

Inspection of the RHS of Φ_t in (5b) likewise fails to deliver an immediate answer for the sign of Φ_t: it is not obvious how the reduction in the payoff $\pi(p, c+t)$ under an optimal deviation in a cartel partner's market (brought about by an increase in t) compares to the increase in the Bertrand-Nash payoff $\pi(p^n, c)$ (associated with an increase in t).[24] One might conjecture that the values of δ, t and p matter here. We will substantiate the validity of the conjecture shortly. In addition, we will demonstrate that the collusive payoff π^m_{ROW} in ROW will be front and center in our characterization of Φ_t and the properties of the ICC.

For reasons that will become obvious shortly, it helps to initially view p as a parameter and ask: for any given (p, t, π^*_{ROW}), what is the value of δ that ensures $\Phi(\cdot) = 0$? One can see, by solving for the value of δ that ensures $\Phi = 0$ in (3), that a solution $\underline{\delta} \in [\frac{1}{2}, 1)$ exists. In particular, we have

$$\underline{\delta} = \underline{\delta}(p, \cdot) \equiv \frac{\Pi^d - \Pi^a}{\Pi^d - \Pi^n} = \frac{1}{2}\frac{\Psi + \Omega}{\Omega} = \frac{\frac{1}{2}\pi^*_{ROW} + \pi(p, c+t)}{\pi^*_{ROW} + \pi(p, c+t) + \pi(p, c) - \pi(p^n, c)}, \tag{6}$$

for $p \in (p^n, p^m]$. As established in Lemma 2(a), $\underline{\delta}$ enables us to rewrite the ICC in (2) as $\delta \geq \underline{\delta}(p, \cdot)$. Thus, the understanding of the dependence of $\underline{\delta}$ on its arguments is key to characterizing the prices in $\mathcal{F}(t, \delta, \pi^*_{ROW})$ that satisfy the cartel's ICC.[25] One can view $\underline{\delta}$ as the minimum discount factor that is capable of sustaining a collusive price p, including p^m.

Lemma 3 below establishes the relationship between $\underline{\delta}$ and p, and the way this relationship is conditioned by trade cost level t and collusive profit π^*_{ROW}. Then, with the help of Proposition 1 that follows Lemma 3, we characterize the dependence of $\underline{\delta}$ on t and π^*_{ROW}. This is a substantive endeavor for two reasons. First, if one focused on $p = p^m$, Proposition 1 would explain how the stability of maximal collusion would depend on trade costs and the profitability in ROW. This analysis is of interest in its own right and is normally the

[24]This is so because $\pi_t(p, c+t) = -D(p)$ (< 0), whereas $\pi_p(p^n, c) = D(p^n) + tD'(p^n)$ (≥ 0) for $t \in [0, \bar{t}]$ (with equality for $t = \bar{t}$) and $D(p) < D(p^n)$ for $p \in (p^n, p^m]$.

[25]For the reasons emphasized earlier, we have replaced τ with π^*_{ROW} in $\mathcal{F}(\cdot)$ as this is a more general treatment of ROW.

ultimate goal of the numerous contributions to the existing literature. However, p^m may be unsustainable. Our forthcoming analysis will identify and characterize the set of efficient prices associated with any $\delta \in (\frac{1}{2}, 1)$. In particular, we will explain how these prices and the welfare levels they give rise to depend on trade costs and profit opportunities in ROW.

Lemma 3 *(The ICC and collusive prices) Suppose $t \in (0, \bar{t})$ and consider a price $p \in (p^n, p^m]$. Then the minimum discount factor $\underline{\delta}$ is increasing in p (i.e., $\underline{\delta}_p > 0$). Moreover, the value of $\lim_{p \to p^n} \underline{\delta}$ depends on π^*_{ROW} as follows:*

a) if $\pi^*_{ROW} = 0$, then $\lim_{p \to p^n} \underline{\delta} = \frac{D(p^n)}{2D(p^n)+(p^n-c)D'(p^n)} \in [\frac{1}{2}, 1]$;

b) if $\pi^*_{ROW} > 0$, then $\lim_{p \to p^n} \underline{\delta} = \frac{1}{2}$.

Lemma 3 establishes an intuitive point: for any given trade cost level t, the higher the collusive price p targeted by the cartel, the higher the associated minimum discount factor and thus the tighter the ICC. In other words, the difficulty of sustaining collusion is increasing in the collusive price level.[26] What is the intuition behind this finding? As can be seen from the definition of $\underline{\delta}$ in (6), there are two channels through which increases in p travel: The first channel involves the effect of p on the representative firm's per-period benefit $\Pi^d - \Pi^a$ due to a defection from the agreement on p (which is the numerator of $\underline{\delta}$ and equals $\frac{1}{2}\pi^*_{ROW} + \pi(p, c+t)$). This effect is increasing in p because $\pi_p(p, c+t) > 0$ for $p < p^m$. The second channel is related to the effect of p on the per-period profit differential $\Pi^d - \Pi^n$ in the punishment phase (i.e., the denominator of $\underline{\delta}$). This effect is also increasing in p because the $\pi(p, c)$ and $\pi(p, c+t)$ components of Π^d are both increasing in p and Π^n remains fixed. Clearly, then, increases in p generate two conflicting effects on $\underline{\delta}$. In the proof in Appendix A, however, we show that (A2) ensures the former effect of raising p dominates the latter, thus causing $\underline{\delta}$ to rise.

Lemma 3 also provides a glimpse into the importance of ROW in the determination of the value of $\underline{\delta}$ as $p \to p^n$. In particular, if profit opportunities in ROW do not exist (i.e., if $\pi^*_{ROW} = 0$), as required in part (a), the minimum discount factor $\underline{\delta}$ assumes a value in the $[\frac{1}{2}, 1]$ interval. The exact value of $\underline{\delta}$ depends on the Bertrand-Nash price which, as noted earlier, depends on trade cost t. Specifically, $\underline{\delta} \to \frac{1}{2}$ for $p \to p^n$ if $t \to 0$, whereas $\underline{\delta} \to 1$ for $p \to p^n$ if $t \to \bar{t}$. On the other hand, if $\pi^*_{ROW} > 0$ (as needed in part (b)), we always have $\underline{\delta} \to \frac{1}{2}$ as $p \to p^n$ regardless of the value of p^n. As we will see shortly, the magnitude of the cartel's profit opportunities in ROW will shape the nature of multimarket collusion.[27]

[26] As a special case, consider the linear demand function $D(p) = \alpha - \beta p$. One can show that, in this case, the minimum discount factor is given by $\underline{\delta} = \frac{\alpha/\beta - p}{2(\alpha/\beta - p - t/2)}$, which is increasing in p. Lemma 3 establishes the validity of this insight for more general demand functions. Naturally, the Bertrand-Nash price p^n and the monopoly price p^m serve as lower and upper bounds to the range of collusive prices, respectively, one could consider.

[27] The proof of Lemma 3 also unveils the following idea: The higher the profitability in ROW, the more

3.2 Collusive Stability

Equipped with Lemmas $1-3$, we now take a deeper look at the dependence of the minimum discount factor $\underline{\delta}$ on trade cost levels t and profits in *ROW* π^*_{ROW}. More specifically, we shed light on how the sustainability of any feasible price agreement depends on t and π^*_{ROW}.

Proposition 1 *(Collusive stability) For any price agreement* $p \in (p^n, p^m]$ *that might be targeted by the cartel and a profitability level* π^*_{ROW} *in ROW, the associated minimum discount factor* $\underline{\delta} = \underline{\delta}(p, t, \pi^*_{ROW})$ *is quasi-concave in trade costs* t *on* $T^x \equiv [0, t_x)$*, where* $t_x = t_x(p) \equiv p - c > 0$*—strictly increasing when* $\pi^*_{ROW} = 0$*, and strictly quasi-concave with an interior peak when* $\pi^*_{ROW} > 0$*. In particular:*

a) If $\pi^*_{ROW} = 0$*, then* $\underline{\delta}_t > 0$ *for* $t \in T^x$*: fragmentation raises the patience requirement of every agreement. Furthermore,*

i) $\lim_{t \to 0} \underline{\delta} = \frac{1}{2}$;

ii) $\underline{\delta}^x(p) \equiv \lim_{t \to t_x} \underline{\delta}(p, t) = \frac{D(p)}{2D(p) + (p-c)D'(p)}$;

iii) $\lim_{p \to c} \underline{\delta}^x = \frac{1}{2}$, $\lim_{p \to p^m} \underline{\delta}^x = 1$ *and* $d\underline{\delta}^x/dp > 0$.

b) If $\pi^*_{ROW} > 0$*, there exists a unique* $t_0 = \arg\max_t \underline{\delta}$ *for* $t \in T^x$*: the patience requirement peaks at an interior trade cost level. Also,*

i) $\lim_{t \to 0} \underline{\delta} = \lim_{t \to t_x} \underline{\delta} = \frac{1}{2}$;

ii) $\partial\underline{\delta}/\partial\pi^*_{ROW} \leq 0$*, with equality for* $t = 0$;

iii) $\partial\underline{\delta}^0/\partial p > 0$*, where* $\underline{\delta}^0 \equiv \underline{\delta}(p, t_0, \pi^*_{ROW})$;

iv) $\underline{\delta}_{\max} = \underline{\delta}^0 = \underline{\delta}^m < 1$ *is the global maximum at* $t_0(p^m, \pi^*_{ROW})$.

The first substantive claim in Proposition 1 is that the minimum discount factor $\underline{\delta}$ is strictly quasi-concave in trade costs t. To deepen our understanding of this relationship, let us consider the effect of raising t on the per-period gain from a deviation $\Pi^d - \Pi^a$ and the per-period profit differential $\Pi^d - \Pi^n$ in (6). The former effect is negative because the increase in t causes the marginal cost of delivering a product to the partner's export market to rise, thus bringing about a reduction in the $\pi(p, c+t)$ of $\Pi^d - \Pi^a$. Moreover, the effect of increasing t on $\Pi^d - \Pi^n$ is also negative because, in addition to the just-described reduction of $\pi(p, c+t)$, which reduces Π^d, the effect of t on Π^n is positive (because $d\Pi^n/dt = \pi^n_p > 0$).[28] Which of these two opposing effects on $\underline{\delta}$ dominates, however, depends on the profitability of exporting to *ROW* π^*_{ROW} and the initial value of trade cost t.

likely that the minimum discount factor $\underline{\delta}$ will be increasing in p. Interestingly, this would happen even if the restriction on the curvature of $D(p)$ in Assumption (A2) were not satisfied.

[28] The sign of the latter effect is positive because higher trade costs make it easier for cartel members to practice limit pricing in their domestic markets, thereby softening the severity of the punishment.

To emphasize the importance of π^*_{ROW} for the stability of collusive outcomes, it helps to once again treat the cases of $\pi^*_{ROW} = 0$ and $\pi^*_{ROW} > 0$ separately; hence, the two parts of Proposition 1. To sharpen intuition, these parts (a) and (b) are supplemented with Figs. 1 and 2, respectively. Panel (a) in both figures depicts the minimum discount factors associated with several potential price agreements. Specifically, the pink schedule $\underline{\delta}^m$ captures the minimum discount factor associated with the monopoly outcome p^m. The blue and green schedules are associated with two other possible collusive prices below p^m (but above p^n).

As noted in part (a) and illustrated in panel (a) of Fig. 1, the various minimum discount factor schedules drawn there are increasing in trade costs t and reach their respective peaks at some point on the black schedule labeled $\underline{\delta}^x$. Schedule $\underline{\delta}^x$ identifies the trade cost and price combinations (along with the associated minimum discount factor values) that ensure $p^n \to p$.[29] Thus, when $\pi^*_{ROW} = 0$, the peak of any minimum discount $\underline{\delta}$ is at $t \to t_x = p - c$ (which ensures that $p^n \to p$).[30] The driving force is that, with $\pi^*_{ROW} = 0$, the (negative) effect of t on $\underline{\delta}$ through the deviation benefit $\Pi^d - \Pi^a$ is dominated by the opposing effect on the punishment cost through $\Pi^d - \Pi^n$—an insight that applies to all feasible collusive prices, not just monopoly pricing.[31]

Turning to part (b), which addresses the case of $\pi^*_{ROW} > 0$ and is depicted in panel (a) of Fig. 2, the key difference from the case of $\pi^*_{ROW} = 0$ considered above is this: for any targeted price $p \in (p^n, p^m]$, there exists a trade cost level $t_0 = t_0\left(p, \pi^*_{ROW}\right) \in T^x$ such that $\underline{\delta}_t > 0$ on $\{t \in T^x : t < t_0\}$ while $\underline{\delta}_t < 0$ on $\{t \in T^x : t > t_0\}$. Thus, $\underline{\delta}$ is strictly quasi-concave in t, with t_0 being the unique maximizer of $\underline{\delta}$. The black schedule $\underline{\delta}^0$ depicted in Fig. 2(a) connects the peaks of the $\underline{\delta}$ schedules that are associated with alternative collusive prices. As such, $\underline{\delta}^0$ identifies the combinations of trade cost levels and prices that place us on these peaks. In the proof to part $(b.iii)$, which is based on Lemma 3 and the implicit function theorem, we prove that $\underline{\delta}^0$ is upward-sloping.[32]

Part $(b.i)$ implies that $\lim_{t\to 0} \underline{\delta} = \frac{1}{2}$, as was the case earlier in part $(a.i)$. Outcomes differ, however, when $t \to t_x$. As can be inferred upon inspection of Fig. 2(a), we now have $\lim_{t\to t_x} \underline{\delta} = \frac{1}{2}$. Thus, the possibility of positive profits in ROW shifts $\lim_{t\to t_x} \underline{\delta}$ from

---

[29] As shown in the proof, to establish the result that $\underline{\delta}^x_t > 0$, we also use the fact that $\underline{\delta}_p > 0$ (Lemma 3).

[30] It is worth noting that $\underline{\delta}$ is not defined at $p = p^n$ because the sums of per period profits in the numerator and the denominator of $\underline{\delta}$ in (6) equal 0. Nonetheless, as described in parts $(a.i)$ and $(a.ii)$, this limit exists and can be obtained with the help of L'Hôpital's rule.

[31] Our proof that $\underline{\delta}_t > 0$ establishes that $\underline{\delta}$ is strictly quasi-concave in t, as stated in Proposition 1. Incidentally, in the special case of linear demand, which attracted the attention of earlier contributions to the literature (e.g., Lommerud and Sørgard, 2001; Akinbosoye et al., 2012), the minimum discount factor associated with $p = p^m$ $(= \frac{\alpha/\beta + c}{2})$ is $\underline{\delta}^m = \frac{\alpha/\beta - c}{2(\alpha/\beta - c - t)}$, which is increasing (and convex) in t.

[32] In the case of linear demand, the peak location $t_0\left(p, \pi^*_{ROW}\right)$ solves a quadratic in t and $\underline{\delta}^0$ admits no compact closed form. As $\pi^*_{ROW} \to 0$, the locus converges to $\underline{\delta}^x$, which for linear demand can be written as $\frac{\alpha/\beta - c - t}{2(\alpha/\beta - c) - 3t}$.

1 (the case in part (a)) to $\frac{1}{2}$ (the case in part (b)). Moreover, as pointed out in part ($b.ii$), the higher the profit in ROW, the lower the minimum discount factor $\underline{\delta}$ associated with any collusive price, including the monopoly price p^m. Therefore, larger π^*_{ROW} values improve the stability of feasible price agreements $p \in (p^n, p^m]$. Intuitively, an increase in π^*_{ROW} raises the per-period punishment cost more than the deviation benefit, causing $\underline{\delta}$ to fall.[33] A noteworthy implication of these properties is that $\underline{\delta}$ attains a global maximum at $t_0\left(p^m, \pi^*_{ROW}\right)$, labeled $\underline{\delta}_{\max}$ (< 1), as noted in part ($b.iv$).

3.3 Efficient Price Agreements

Having explored the requirements on the discount factor to sustain potential price agreements in $(p^n, p^m]$, we are now in a position to address the following questions: How are efficient price agreements determined? And how do such agreements depend on the value of the actual discount factor δ, trade costs t, and the profitability of exporting to ROW? We have already seen that $p^* = p^n$ for $\delta \in (0, \frac{1}{2}]$. So, in what follows, we focus on $\delta \in \left(\frac{1}{2}, 1\right)$ and, for clarity, we organize our presentation on the basis of whether $\pi^*_{ROW} = 0$ or $\pi^*_{ROW} > 0$.

Starting with $\pi^*_{ROW} = 0$, by part (a) of Proposition 1, we know that the minimum discount factor that is capable of sustaining the monopoly price p^m, which we label $\underline{\delta}^m$, and schedule $\underline{\delta}^x$ (which is the set of the largest minimum discount factors obtained when, for any given price, we let $t \to t_x$) are continuously increasing in t from $\frac{1}{2}$ to 1. Now select an arbitrary actual discount factor $\delta \in \left(\frac{1}{2}, 1\right)$ and consider successive increases in trade costs t from 0 to $\bar{t}$, as indicated by the horizontal dashed line in Fig. 1(a). This line necessarily intersects $\underline{\delta}^m$ and $\underline{\delta}^x$ at trade cost levels $t_1(\delta)$ and $t_2(\delta)$, respectively. These trade cost levels, which are unique and increasing in δ, divide the interval $[0, \bar{t}]$ into the following three (and distinct) subintervals: $[0, t_1]$, (t_1, t_2), and $[t_2, \bar{t}]$.

Since $\underline{\delta}(p, t) \leq \underline{\delta}(p^m, t) = \underline{\delta}^m(t) \leq \delta$ for all $t \in [0, t_1]$ (with equality at $t = t_1$) and all prices $p \in (p^n, p^m]$ by Lemma 3, the monopoly price is sustainable there, and $p^* = p^m$ follows because Π^a is increasing in p. Next, note that the monopoly price p^m cannot be sustained for $t > t_1$. Moreover, if trade costs are large enough so that $t \in [t_2, \bar{t}]$, we have $\underline{\delta}^x \leq \delta$. Thus, in this case, there is no price other than p^n that satisfies the ICC, so $p^* = p^n$ $(= c + t)$, with p^* being an increasing function of t.

Let us now consider trade cost values in (t_1, t_2) which is the interval associated with the shaded region in Fig. 1(a). The distinguishing feature of this set of trade cost values is that it implies that the ICC is active (i.e., it holds with equality) and, as such, defines implicitly a price p^c that solves $\underline{\delta}(p^c, t) = \delta$ ("c" for "constrained"); therefore, $p^* = p^c$ in this case.

[33] Although this point is not shown in Fig. 2(a), it could be visualized by flattening the $\underline{\delta}$ curves via imaginary strokes on their peaks. In the limit, when $\pi^*_{ROW} \to \infty$, $\underline{\delta}$ converges to the horizontal line at $\underline{\delta} = \frac{1}{2}$.

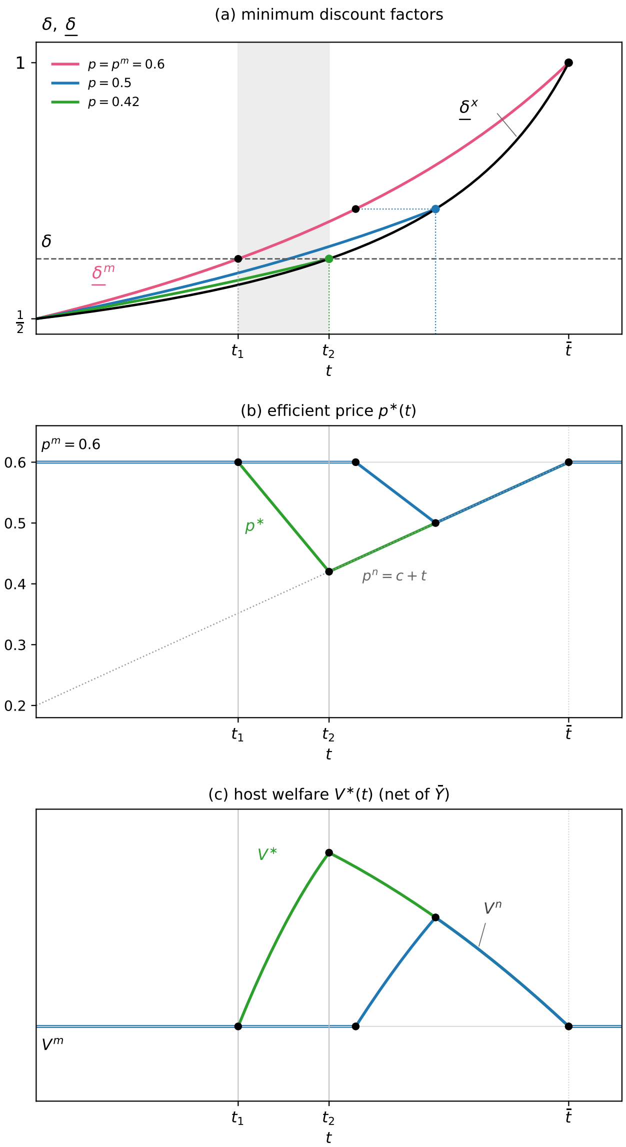

Figure 1: The ICC, efficient prices, and welfare under domestic monopoly, $\pi^*_{ROW} = 0$: (a) minimum discount factors $\underline{\delta}(p,t)$ for the monopoly price p^m (pink) and two lower collusive prices (blue, green), together with the locus $\underline{\delta}^x$ (black); the horizontal dashed line marks the actual discount factor δ, and the shaded band the interval (t_1, t_2) on which the ICC binds; (b) the efficient price $p^*(t)$ of Proposition 2; (c) host welfare $V^*(t)$, net of $\bar{Y}$. Panels (b)–(c) trace the efficient agreement for two discount factors, $\delta = 0.617$ (green path) and $\delta' = 0.714$ (blue path); each constrained path terminates on $\underline{\delta}^x$ at its junction price (dots)—0.42 and 0.5, the two lower prices of panel (a)—beyond which welfare declines along the Nash locus $V^n(t)$. Linear-demand illustration: $D(p) = 1 - p$, $c = 0.2$, $n = 1$; beyond $\bar{t} = 0.4$ (shown marginally) the efficient outcome is autarky monopoly. The collapse threshold t_2 (marked in panel (a), where δ meets $\underline{\delta}^x$ so that $p^* = p^n$) is the $\pi^*_{ROW} \to 0$ limit of the price-trough location t_0 of Fig. 2(b) (cf. Appendix B).

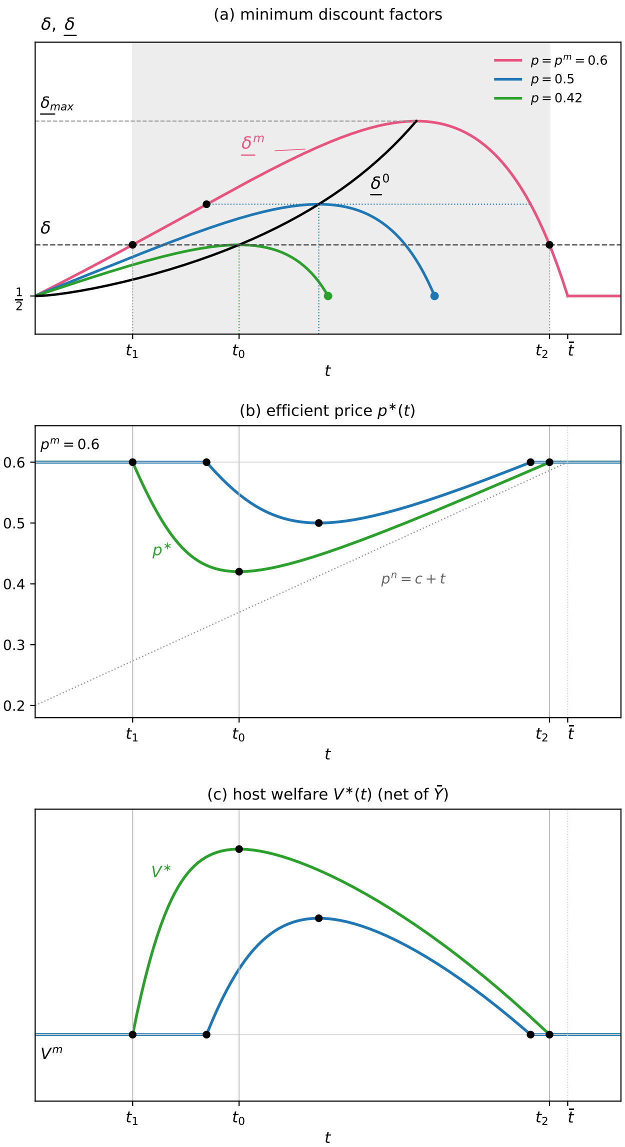

Figure 2: The ICC, efficient prices, and welfare under domestic monopoly, $\pi^*_{ROW} > 0$: (a) minimum discount factors $\underline{\delta}(p, t, \pi^*_{ROW})$, now hump-shaped in t, for the monopoly price p^m (pink) and two lower collusive prices (blue, green), together with the locus $\underline{\delta}^0$ of their peaks (black); the horizontal dashed line marks δ, and the shaded band the interval (t_1, t_2) on which the ICC binds; (b) the efficient price $p^*(t)$ of Proposition 3, U-shaped on (t_1, t_2) with minimum at t_0; (c) host welfare $V^*(t)$, net of $\bar{Y}$, hump-shaped with peak at t_0. Panels (b)–(c) trace the efficient agreement for $\delta = 0.540$ (green path) and $\delta' = 0.572$ (blue path), the peak values of the green and blue schedules; each constrained path attains its minimum price on the locus $\underline{\delta}^0$, and each lower-price schedule terminates at $\underline{\delta} = \frac{1}{2}$ at $t = p - c$ (dots). Linear-demand illustration: $D(p) = 1 - p$, $c = 0.2$, $n = 1$, $\pi^*_{ROW} = 0.06$; for $\delta = 0.540$ the collapse-side threshold is $t_2 = 0.386$, close to $\bar{t}$ (see footnote in Section 4); beyond $\bar{t}$ (shown marginally), autarky monopoly obtains.

But $\underline{\delta}_p > 0$ by Lemma 3, and $\underline{\delta}_t > 0$ by Proposition 1(a). Therefore, $dp^c/dt < 0$ for any $t \in (t_1, t_2)$.

The above ideas can be visualized with the help of Fig. 1(b), which is related to panel Fig. 1(a) above it. Note how p^* depends on trade costs t. For low t values, p^* equals p^m and remains constant at this level as t rises until t reaches t_1. At that point, $p^* = p^c$ and falls with increases in t to attain a minimum at t_2. From there on, $p^* = p^n$ and p^* rises with increases in t (because $p^n = c + t$). For additional clarity, we summarize these ideas in Proposition 2 below.

Proposition 2 *(Efficient prices for $\pi^*_{ROW} = 0$) Suppose $\pi^*_{ROW} = 0$ and $\delta \in (\frac{1}{2}, 1)$. Then, there exist trade cost levels $t_1(\delta)$ and $t_2(\delta)$, such that: (a) t_1 solves $\delta = \underline{\delta}(p^m, t_1)$; (b) t_2 solves $\delta = \underline{\delta}^x(p^n(t_2))$; and (c) $0 < t_1 < t_2 < \bar{t}$ with $t_1'(\delta) > 0$ and $t_2'(\delta) > 0$. The efficient price p^* that solves the cartel's optimization problem is then described by*

$$p^*(t, \delta) = \begin{cases} p^m & \text{if} \quad t \in [0, t_1] \\ p^c & \text{if} \quad t \in (t_1, t_2) \\ p^n & \text{if} \quad t \in [t_2, \bar{t}] \end{cases}$$

where $p^c = p^c(\delta, t)$ solves $\delta = \underline{\delta}(p^c, t)$ and satisfies: $p^c_t < 0$, $p^c_\delta > 0$.

In words: an unconstrained cartel sustains the monopoly price at low trade costs; over an intermediate range, fragmentation forces the collusive price down; and at high trade costs the cartel is reduced to the Bertrand-Nash outcome, which itself rises with t.

Proof: The construction preceding the proposition establishes the regime characterization. The comparative statics follow from the implicit function theorem: $t_1'(\delta) > 0$ and $t_2'(\delta) > 0$ from the strict monotonicity of $\underline{\delta}^m$ and $\underline{\delta}^x$ in t (Proposition 1(a)), and $p^c_t < 0$, $p^c_\delta > 0$ from $\underline{\delta}_p > 0$ (Lemma 3) together with $\underline{\delta}_t > 0$ (Proposition 1(a)). ||

Proposition 2 provides a complete characterization of efficient price agreements for cartels facing general demand functions. The thresholds have a simple reading: t_1 is the trade cost at which the ICC starts to bind, and t_2 is the *collapse threshold*: the trade cost at which the highest sustainable collusive price has fallen all the way to the Bertrand-Nash level p^n. At t_2 the collusive premium $p^c - p^n$ vanishes; for $t > t_2$ no price above p^n is sustainable, so collusion collapses to the static Bertrand-Nash outcome (whence the name). Equivalently, t_2 is the trade cost at which the actual discount factor δ meets the schedule $\underline{\delta}^x$—the black locus in Fig. 1(a)—below which even a price just above p^n cannot be sustained. Put differently, efficient price agreements are non-monotone in the trade costs that separate the cartel hosts: when trade costs rise from low levels, fragmentation eventually becomes anti-collusive, and on (t_1, t_2) it forces the cartel to settle for prices further below the monopoly level.

How might the efficient price agreement change if cartel members had an opportunity to export their products to *ROW*? And how might the level of profitability in *ROW* matter? To address these questions recall from part (b) of Proposition 1 that $\underline{\delta}$ attains a maximum at t_0 and that $\underline{\delta}_{\max} \equiv \underline{\delta}\left(p^m, t_0\left(p^m, \pi^*_{ROW}\right), \pi^*_{ROW}\right)$ where

$$\frac{1}{2} < \underline{\delta}_{\max} < 1, \quad \lim_{\pi^*_{ROW} \to 0} \underline{\delta}_{\max} = 1, \quad \lim_{\pi^*_{ROW} \to \infty} \underline{\delta}_{\max} = \frac{1}{2}, \text{ and } d\underline{\delta}_{\max}/d\pi^*_{ROW} < 0.$$

The properties of $\underline{\delta}_{\max}$ described above follow readily from our analysis of minimum discount factors in Proposition 1(b). As noted in Proposition 3 below, a noteworthy (and previously unobserved) feature of $\underline{\delta}_{\max}$ is that it implies $p^* = p^m$ for all $t \in [0, \bar{t}]$ if $\delta \in [\underline{\delta}_{\max}, 1)$. Matters are a bit more nuanced, however, when $\delta \in (\frac{1}{2}, \underline{\delta}_{\max})$. Proposition 3 explains.

Proposition 3 *(Efficient prices for $\pi^*_{ROW} > 0$) Suppose $\pi^*_{ROW} > 0$ and $\delta \in (\frac{1}{2}, 1)$.*

a) If $\delta \in [\underline{\delta}_{\max}, 1)$, then $p^* = p^m$ for all $t \in [0, \bar{t}]$.

b) If $\delta \in (\frac{1}{2}, \underline{\delta}_{\max})$, then

$$p^*\left(t, \delta, \pi^*_{ROW}\right) = \begin{cases} p^m & \text{if} \quad t \in [0, t_1] \cup [t_2, \bar{t}] \\ p^c & \text{if} \quad t \in (t_1, t_2) \end{cases}$$

where t_1 and t_2 solve $\delta = \underline{\delta}\left(p^m, t_i, \pi^*_{ROW}\right)$ for $i = 1, 2$, $p^c = p^c(\cdot)$ solves $\delta = \underline{\delta}\left(p^c, t, \pi^*_{ROW}\right)$, and

i) $0 < t_1 < t_2 < \bar{t}$, $\partial t_1/\partial \delta > 0$, and $\partial t_2/\partial \delta < 0$;

ii) $\partial p^c/\partial t \lesseqqgtr 0$ as $t \lesseqqgtr t_0$ for $t \in (t_1, t_2)$, where $t_0 \equiv t_0\left(p_0, \pi^*_{ROW}\right)$ and p_0 is the unique price solving $\underline{\delta}^0\left(p_0\right) = \delta$;

iii) $\partial p^c/\partial \delta > 0$ and $\partial p^c/\partial \pi^*_{ROW} > 0$.

In words: profitable exports to *ROW* act as collateral for collusion at home, sustaining monopoly pricing at low and at high trade costs alike; only at intermediate trade costs does the ICC bind, and there the collusive price traces the mirror image of the hump in the patience requirement.

The two parts of Proposition 3 can be visualized with the help of Fig. 2(b). Part (a) is straightforward: When the cartel can obtain positive profits in *ROW*, the cartel may sustain the most collusive price p^m for all trade cost levels if δ exceeds $\underline{\delta}_{\max}$. But the cartel may also sustain p^m even if $\delta \in (\frac{1}{2}, \underline{\delta}_{\max})$. As explained in part (b) this possibility crucially depends on the level of trade costs t. As in Proposition 2, there exist threshold trade cost levels t_1 and t_2. The difference is that now $\delta = \underline{\delta}(p^m, t_i, \pi^*_{ROW})$ admits two solutions t_1 and t_2 $(i = 1, 2)$. The two thresholds now simply bracket the interval of trade costs over which the ICC binds. Because there is slack in the ICC for $p = p^m$ when $t \in [0, t_1] \cup [t_2, \bar{t}]$, the efficient price agreement coincides with the monopoly price (i.e., $p^* = p^m$) if t is sufficiently low or sufficiently high.[34] On the other hand, if trade costs t assume intermediate values on the interval (t_1, t_2), the ICC is active and requires p^* to fall short of p^m. Specifically, $p^* = p^c$ in this case. Parts (*b.ii*) and (*b.iii*) follow readily from our earlier analysis. Part (*b.ii*) reveals that increases in trade costs ($t \uparrow$) are pro-collusive for initial values in (t_0, t_2) but anti-collusive for $t \in (t_1, t_0)$. Part (*b.iii*) points out that increases in δ or in π^*_{ROW} are unambiguously pro-collusive.

Finally, it is useful to note that Proposition 3 suggests (and Fig. 2(b) confirms) that $p^* > p^n$ for all discount factor and trade cost values considered.[35]

4 Welfare under Domestic Monopoly

In this section, we examine the welfare implications of changes in trade costs t—with an emphasis on increases in t, i.e., fragmentation—and of expanded profit opportunities in *ROW* for cartel hosts. In this partial equilibrium setting, social welfare in the representative cartel host may be identified with the indirect utility $V = V(p, Y) = v(p) + Y$, where $v'(p) = -D(p)$ (Roy's identity) and $Y = \bar{Y} + \pi(p, c) + \frac{1}{2}\pi^*_{ROW}$ is national income: $\bar{Y}$ captures income from factor payments and is constant; and $\pi(p, c)$ and π^*_{ROW} are the representative cartel member's profit obtained domestically and in *ROW*, respectively. For later reference, write $V^m \equiv V(p^m, \cdot)$ for host welfare at the monopoly price and $V^n(t) \equiv V(c + t, \cdot)$ for host welfare along the Bertrand-Nash locus. Since $p^n = c + t$ and $\bar{t} = p^m - c$, the two coincide at $t = \bar{t}$: at prohibitive trade costs each host is an isolated monopoly.

Total differentiation of V gives

$$dV = v'(p)\, dp + \pi_p dp + \frac{1}{2} d\pi^*_{ROW} = (p - c)\, D'(p)\, dp + \frac{1}{2} d\pi^*_{ROW}. \tag{7}$$

The middle expression in (7) decomposes the welfare change into: (i) the reduction in consumer surplus due to a price increase ($dp > 0$); (ii) the increase in income due to an increase in the cartel member's profit at home (provided $\pi_p > 0$); and (iii) the change in income due to a change in the cartel profit in *ROW*. The first term of the right-most expression, $(p - c)\, D'(p)\, dp$, consolidates effects (i) and (ii): it captures the deadweight loss

[34] It might appear that the interval of trade costs $[t_2, \bar{t}]$ in Fig. 2(a) is empty but, as we have seen by the construction of t_2, this is not true.

[35] This contrasts with the case of $\pi^*_{ROW} = 0$ studied in Proposition 2 where $p^* = p^n$ for sufficiently high t values.

due to a price increase. This effect is negative because $D'(p) < 0$ and the domestic price p exceeds marginal cost c.

Focusing on efficient prices p^* (so that $V = V^*$ and $dp = dp^*$) it becomes clear from (7) that, when p^* adjusts due to changes in the values of trade costs t or the discount factor that were identified in Propositions 2 and 3, welfare V^* moves in the opposite direction to the change in p^*. Fig. 1(c) depicts the dependence of V^* on trade costs t when $\pi^*_{ROW} = 0$ (and $d\pi^*_{ROW} = 0$ in the background). Reading the figure in the direction of rising trade costs: for t below t_1 the ICC is slack, so $p^* = p^m$ and $V^* = V^m$ are invariant to t. As t rises beyond t_1, fragmentation undermines collusion ($p^* \downarrow$) and welfare rises, peaking at t_2. Beyond t_2 the cartel is inoperative, but the Bertrand-Nash price $p^n = c + t$ itself rises with t, so welfare falls, returning to V^m as $t \to \bar{t}$.

Fig. 2(c) illustrates the relationship between welfare V^* and trade costs t for $\pi^*_{ROW} > 0$. The key difference now is that the cartel is able to sustain some collusion for all non-prohibitive trade cost levels, so $p^* > p^n$. For $\delta \in (\frac{1}{2}, \underline{\delta}_{\max})$, if t lies below t_1, we have $p^* = p^m$ and $V^* = V^m$, and welfare is unaffected by marginal fragmentation. As t rises on (t_1, t_0), fragmentation undermines collusion and welfare rises. However, this is true only until the threshold level t_0 is reached: because further increases in t on (t_0, t_2) facilitate collusion, fragmentation affects welfare adversely there. Once t crosses t_2, the monopoly outcome is restored and welfare remains flat at $V^* = V^m$.[36]

For clarity and easy reference, we may summarize the above insights as follows.

Proposition 4 *(Trade costs and cartel host welfare V^*) A cartel host's welfare V^* is quasi-concave in trade costs t on $[0, \bar{t}]$ and never falls below its monopoly level: $V^*(t) \geq V^m$ throughout, with equality exactly on $[0, t_1] \cup \{\bar{t}\}$ when $\pi^*_{ROW} = 0$ and exactly on $[0, t_1] \cup [t_2, \bar{t}]$ when $\pi^*_{ROW} > 0$. Thus adjustments in trade costs t may affect the welfare of cartel hosts even though there is no cross-hauling in equilibrium.*

a) A necessary condition for increases in trade costs ($t \uparrow$) to benefit cartel hosts ($V^* \uparrow$) is that t lie in (t_1, t_2) when $\pi^*_{ROW} = 0$, and in (t_1, t_0) when $\pi^*_{ROW} > 0$; in either case the initial level of t must be moderately low.

b) Cartel hosts weakly prefer any degree of fragmentation ($t > 0$) to free intra-regional trade ($t = 0$), and strictly prefer a moderate degree of it. When $\pi^*_{ROW} = 0$, moreover, $V^*(0) = V^*(\bar{t}) = V^m$: hosts are indifferent between frictionless intra-regional trade and prohibitive trade costs, and strictly prefer any intermediate level. Only intermediate barriers discipline the cartel.

In words: fragmentation benefits cartel hosts only in moderation. Hosts are never worse off

[36]This point may be unclear in Fig. 2(c) because t_2 is close to $\bar{t}$ for the value of δ considered there. However, it can be inferred from Fig. 2(a).

than under monopoly pricing, gain most at the trade cost that brings the cartel to the brink of collapse, and—absent profit opportunities in ROW—are indifferent between frictionless intra-regional trade and prohibitive barriers.

Proof: Both parts follow from (7) (with $d\pi^*_{ROW} = 0$ here) together with Propositions 2 and 3. Since dV^* and dp^* have opposite signs, V^* rises only where p^* falls—on (t_1, t_2) when $\pi^*_{ROW} = 0$ and on (t_1, t_0) when $\pi^*_{ROW} > 0$—which proves part (a) and, together with the flat and declining segments identified in Propositions 2 and 3, the quasi-concavity asserted in the statement. For the level comparisons, (7) implies that V is strictly decreasing in p on $(c, p^m]$, while $p^* \leq p^m$ everywhere; hence $V^* \geq V^m$, with equality if and only if $p^* = p^m$. By Propositions 2 and 3 this occurs on $[0, t_1]$ and, when $\pi^*_{ROW} > 0$, on $[t_2, \bar{t}]$. When $\pi^*_{ROW} = 0$ and $t = \bar{t}$ we have $p^* = p^n = c + \bar{t} = p^m$, so $V^*(\bar{t}) = V^m = V^*(0)$, while $V^* > V^m$ on $(t_1, \bar{t})$. This proves part (b). ||

One could also ask how trade cost adjustments affect global efficiency. In our context, the answer is clear.

Remark 2 *(Host and global welfare) Consider changes in trade costs t. Because the cartel extracts monopoly rents from ROW at every t in all the cases considered in this section, $p^*_{ROW} = p^m_{ROW}$ is invariant to t, so ROW's surplus is unaffected and global welfare moves in the same direction as the welfare of the cartel hosts. The alignment is specific to comparative statics in t: an improvement in ROW profit opportunities brought about by lower external trade costs τ also lowers p^m_{ROW} and thus raises ROW's surplus, so it may raise global welfare while lowering host welfare (Proposition 5). As we show in Section 5 (Proposition 8(c)), the alignment in t itself breaks down under domestic competition.*

Remark 3 *(Detection: prices, not trade volumes) Along the entire path of efficient agreements—for every t, δ, and π^*_{ROW}, and under both market structures studied in this article—cartel members never trade with each other: collusion involves geographic withdrawal, and the punishment phase involves limit pricing ($n = 1$) or marginal-cost pricing ($n \geq 2$), neither of which generates imports. Intra-regional trade volumes are therefore identically zero along the entire path, whatever the trade cost, the discount factor, or the profitability of ROW. No trade-based statistic can separate collusion from competition, or a strengthening cartel from a weakening one, and inferences that link market integration or competitive conduct to observed trade flows have no bite here. The identifying information resides in prices: under domestic monopoly the collusive price responds non-monotonically to trade costs (Propositions 2 and 3), whereas under domestic competition it weakly rises with them (Proposition 7 in Section 5). Price-based screens—conditioned on the domestic* *market structure of the industry in question—are thus the appropriate detection instruments. The force of this observation does not rest on volumes being exactly zero. What the model isolates is that trade costs act on collusion through the incentive constraint rather than through shipments; wherever that is so, neither the level nor the direction of change of trade flows is informative about the collusive margin, even where trade is positive. Establishing this in settings with differentiated products, a competitive fringe, or the cross-hauling that arises under quantity competition lies beyond the scope of this article.*

How do (exogenous) improvements in profit opportunities in *ROW* ($\pi^*_{ROW} \uparrow$) affect cartel host welfare? Inspection of (7) reveals that the direct effect of an increase in π^*_{ROW} on V^* is positive because it expands national income. However, as emphasized in part (c) of Proposition 3, the increase in π^*_{ROW} may facilitate collusion and raise p^*. This generates an interesting and, to our knowledge, hitherto unexplored trade-off. Can the latter (potentially pro-collusive) effect overshadow the former (and positive income) effect, so that expansions in profit opportunities in *ROW* reduce welfare of cartel hosts?

We address this question next. Because price adjustments depend on the discount factor δ and on the level of trade costs t, the answer appears intractable at first sight. It is not. The two forces at work can be signed under general demand: for any admissible demand under Assumption (A2), an increase in π^*_{ROW} raises national income directly *but* also reduces it indirectly—by raising the constrained collusive price p^c (Proposition 3($b.iii$))—so immiserization is a question of which force dominates. Proposition 5 answers it at that level of generality; Proposition 6 then specializes to linear demand, where closed forms deliver the full shape of the welfare surface shown in Fig. 3.

Proposition 5 *(Profitability in ROW and welfare V^*) Suppose $\delta \in \left(\frac{1}{2}, \underline{\delta}_{\max}\right)$ and $t \in (t_1, t_2)$ (evaluated at the initial π^*_{ROW}). Then, for* any *demand satisfying (A2), improvements in profit opportunities in ROW ($\pi^*_{ROW} \uparrow$)*

a) reduce welfare V^* if the initial level of π^*_{ROW} is sufficiently low and t is sufficiently close to the collapse threshold $t_2^0 \equiv t_2|_{\pi^*_{ROW}=0}$ of Proposition 2; if in addition demand is weakly convex on $[p^n, p^m]$ (as it is under constant elasticity), this holds at every *$t \in (t_1, t_2)$;*

b) raise welfare V^* if the initial level of π^*_{ROW} is sufficiently high, and this part requires no curvature restriction beyond (A2).

In words: profitable access to third markets is collateral for collusion at home, and paying in that coin can leave the cartel's hosts worse off. When *ROW* is barely profitable, a small improvement there buys a large increase in the collusive price, and the deadweight loss it creates swamps the additional income.

Proof: Part (a) is proved in Appendix B: B.1 establishes the local statement for general demand and B.2 its global completion under weak convexity. Part (b) follows from Proposition 1$(b.ii)$ and Proposition 3: once π^*_{ROW} is sufficiently large the ICC is slack, so $p^* = p^m$, the adverse pro-collusive channel vanishes, and only the direct (and positive) income effect of π^*_{ROW} remains. $\quad ||$

Two features of Proposition 5 deserve emphasis. First, the threshold t_2^0 that localizes part (a) lies inside the constrained interval. It is the $\pi^*_{ROW} \to 0$ limit of the price-trough location t_0 of Proposition 3$(b.ii)$ (Fig. 2(b))—not of the larger trade cost at which the ICC slackens—so $t_2^0 \in (t_1, t_2)$, and the mechanism operates near the trough of the constrained path. Second, the role of demand curvature is not to make the result possible but to make it global. The immiserization region is nonempty under any admissible demand; weak convexity—the very property that a maintained assumption of profit concavity would exclude—extends it to the whole of (t_1, t_2) (Theorem B1 in Appendix B). Failures require strictly concave demand and, as we show there, arise only at low trade costs.

Linear demand deserves separate statement because it delivers closed forms, and with them the complete shape of the welfare surface.

Proposition 6 *(Profitability in ROW and welfare: the linear benchmark) Suppose* $\delta \in \left(\frac{1}{2}, \underline{\delta}_{\max}\right)$ *and* $t \in (t_1, t_2)$ *(evaluated at the initial* π^*_{ROW}*). Moreover, suppose the demand function in the home countries of the cartel takes the linear form:* $D(p) = \alpha - \beta p$ $(\alpha, \beta > 0)$*. Then, part* (a) *of Proposition* 5 *is operative at every* $t \in (t_1, t_2)$*. Moreover, improvements in profit opportunities in ROW* $(\pi^*_{ROW} \uparrow)$

a) reduce welfare V^* *if the initial level of* π^*_{ROW} *is sufficiently low;*

b) raise welfare V^* *if the initial level of* π^*_{ROW} *is sufficiently high,*

so that V^* *is U-shaped in* π^*_{ROW}*. The closed forms for the peak of the constrained path are recorded in Section S3 of the Supplementary Appendix, and the resulting welfare surface is shown in Fig. 3.*

Clearly, then, access to *ROW* and the magnitude of collusive profits obtained there matter for welfare in cartel hosts. In the proof of part (a), we show that the adverse pro-collusive effect due to increases in π^*_{ROW} outweighs its direct (and positive) welfare effect when π^*_{ROW} is small initially. This is so because $p^* - c$ in (7) remains bounded away from zero while the effect on price p^* becomes large when π^*_{ROW} is small. The intuition behind part (b) is simpler. When π^*_{ROW} becomes sufficiently large the adverse pro-collusive effect of π^*_{ROW} on V^* vanishes because we always have $p^* = p^m$ which leaves operational only the direct and positive income effect on welfare. Fig. 3 can help visualize the simultaneous dependence of welfare V^* on π^*_{ROW} and trade costs t. The figure shows that, for given trade

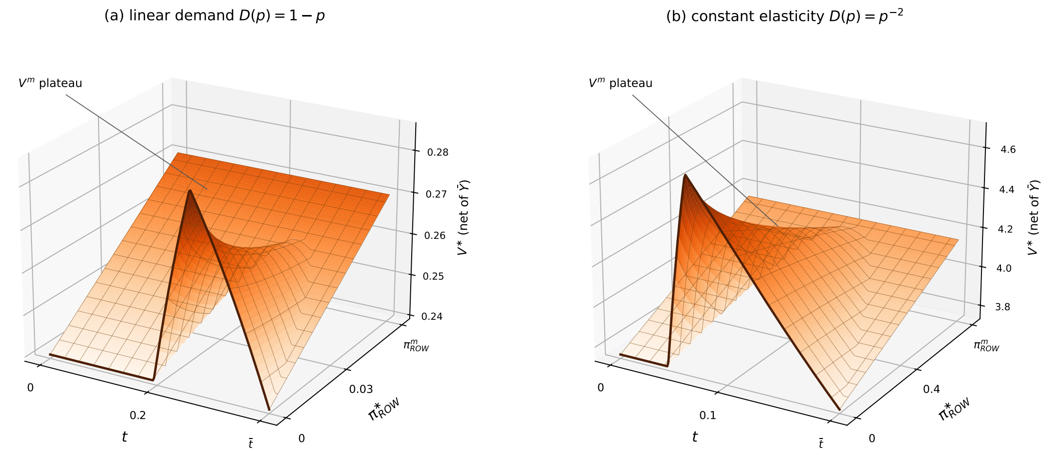

Figure 3: Welfare as a function of trade costs and profits in ROW: host welfare V^* (net of $\bar{Y}$) over $(t, \pi^*_{ROW}) \in [0, \bar{t}] \times [0, \pi^m_{ROW}]$, under (a) linear and (b) constant-elasticity demand. On the flat plateau the ICC is slack and $p^* = p^m$, so $V^* = V^m$ rises linearly in π^*_{ROW}; on the constrained region, V^* is U-shaped in π^*_{ROW} for given t (Proposition 5; Proposition 6 for the linear panel), producing the valley–ridge saddle. Panel (a): $D(p) = 1 - p$, $c = 0.2$, $n = 1$, $\pi^m_{ROW} = 0.06$, $\delta = 0.66$. Panel (b): $D(p) = p^{-2}$, $c = 0.2$, $n = 1$, $\pi^m_{ROW} = 0.80$, $\delta = 0.66$. Panel (a) spans exactly the range of the two preceding figures (but differs in that $\delta = 0.66$): the heavy curve on the front edge captures the welfare path of Proposition 2 drawn in Fig. 1(c) for $\pi^*_{ROW} = 0$; the edge further back captures the case displayed in Fig. 2(c) for $\pi^*_{ROW} = 0.06$. The premise $\frac{1}{2} < \delta < \underline{\delta}_{\max}$ of Proposition 5 is evaluated at the *initial* π^*_{ROW}, and $\underline{\delta}_{\max} \to 1$ as $\pi^*_{ROW} \to 0$ (Section 3), so it holds throughout the low-π^*_{ROW} region on which part (a) operates; δ exceeds $\underline{\delta}_{\max}$ evaluated at π^m_{ROW}, which is what leaves the ICC slack along the far edge and makes part (b) visible, and which illustrates directly that $\underline{\delta}$ is decreasing in π^*_{ROW} (Proposition 1($b.ii$)). Host welfare is strictly decreasing in π^*_{ROW} on about 25% of the domain in panel (a) and about 48% in panel (b). The saddle survives under constant elasticity—a convex, workhorse demand that a profit-concavity assumption would exclude—while the more elastic demand narrows the constrained window, so the valley sits at lower t (cf. the closed form $t_2 = c(2\delta - 1)/[1 + \delta(\varepsilon - 2)]$, decreasing in ε; Appendix B).

costs at which the ICC is binding, welfare is U-shaped in π^*_{ROW}; in contrast, when the ICC is slack, welfare rises linearly in π^*_{ROW}. The figure also illustrates the hump-shaped nature of welfare in trade costs t, for given π^*_{ROW}, studied in Proposition 4. In sum, V^* has the shape of a saddle.

Profit opportunities in ROW may expand for various reasons, including multilateral (external) trade liberalization, technological advances that reduce the costs of shipping merchandise there, or just growth in demand. A noteworthy insight of Proposition 5 is that, in the absence of appropriate regulation, such developments may immiserize cartel hosts—and, by Theorem B1, they do so throughout the constrained range whenever demand is weakly convex, the constant-elasticity case included.

5 Collusion under Domestic Competition

We now examine how outcomes change when each cartel host is home to $n \geq 2$ firms. Two features of the environment change relative to the benchmark of Sections 3 and 4, and both matter fundamentally. First, the punishment phase: with $n \geq 2$ equal-cost local firms in each market, reversion to the Bertrand-Nash equilibrium yields $p^n = c$ and zero profits everywhere, *independently* of trade costs. The limit-pricing channel—through which higher trade costs softened punishment in the benchmark—vanishes. Second, the deviation: because local rivals share demand under a collusive agreement, a deviating firm now undercuts the collusive price not just in *ROW* but also in its *own* market, capturing $D(p)$ rather than $D(p)/n$ there.

5.1 Payoffs and the ICC

Under a collusive agreement, the representative firm earns $\Pi^a = \frac{1}{n}\pi(p,c) + \frac{1}{2n}\pi_{ROW}$, where $\pi_{ROW} \equiv \pi_{ROW}(p_{ROW}, c+\tau) \in [0, \pi^m_{ROW}]$ denotes the per-period cartel profit in *ROW* implied by the agreed price p_{ROW}. Under an optimal deviation the firm undercuts in all markets, so $\Pi^d \approx \pi(p,c) + \pi(p,c+t) + \pi_{ROW}$—an expression formally identical to (1b), though its first term now reflects undercutting at home rather than adherence to the agreement there.[37] Finally, $\Pi^n = 0$, forcing the ICC in (2) to become $\Phi = \Pi^a - (1-\delta)\Pi^d \geq 0$.

It is worth reflecting on why the deviation payoff does not change relative to the benchmark while everything else does. Under domestic monopoly, a deviating firm leaves its home price at the collusive level—it already serves the entire home market—and earns $\pi(p,c)$ there; under domestic competition, a deviator instead undercuts marginally at home and captures the entire market at essentially the same price, again earning $\pi(p,c)$. The two behaviors differ, but their implied payoffs coincide. What changes is the rest of the comparison: the collusive payoff shrinks to a share of $\frac{1}{n}$ in each market while the punishment payoff drops to zero. Domestic competition thus tightens the ICC exclusively through the agreement and punishment sides—a fact that drives every reversal below.

An immediate but consequential observation is that

$$\frac{\partial \Phi}{\partial \pi_{ROW}} = \frac{1}{2n} - (1-\delta), \tag{8}$$

which is *negative* whenever $\delta < 1 - \frac{1}{2n}$. In contrast to the benchmark—where profitable access to *ROW* always relaxed the ICC—higher profits in *ROW* may now tighten it. It

[37] As in Section 2, $\pi(p, c+t) = 0$ whenever $p \leq c+t$, since undercutting in the partner's market is then unprofitable. We refer to prices $p \in (c, c+t]$ as lying in the *home-deviation region* and to prices $p > c+t$ as lying in the *export-deviation region*.

follows that we can no longer assert $p^*_{ROW} = p^m_{ROW}$ at the outset; the cartel's price in *ROW* must be solved for jointly with p. The cartel's instruments are thus the two prices (p, p_{ROW}). Because payoffs depend on p_{ROW} only through $\pi_{ROW}(p_{ROW}, c+\tau)$, which is strictly increasing on $[c+\tau, p^m_{ROW}]$ (Lemma A1 applied to the *ROW* market), it is convenient to treat the profit level $\pi_{ROW} \in [0, \pi^m_{ROW}]$ as the choice variable; every such level is implemented by a unique *ROW* price in $[c+\tau, p^m_{ROW}]$.[38] Lemma 4 collects the properties of the ICC that render this problem tractable. Throughout, we denote with $\kappa \equiv 1 - \frac{1}{2n}$ the classic Bertrand threshold for $2n$ symmetric firms, and with $\Omega \equiv \Pi^d$ the deviation payoff.

Lemma 4 *(The ICC under domestic competition) Suppose $n \geq 2$, $t \in (0, \bar{t})$, $p \in (c, p^m]$, $\pi_{ROW} \in [0, \pi^m_{ROW}]$ and keep in mind that $\kappa \equiv 1 - \frac{1}{2n}$. Then:*

a) $\Phi = (\delta - \underline{\delta})\,\Pi^d$, where $\underline{\delta} = \kappa - \frac{tD(p)}{2n\,\Omega}$ on the export-deviation region and $\underline{\delta} = \kappa - \frac{\pi(p,c)}{2n\,\Omega}$ on the home-deviation region; the two expressions coincide at the junction $p = c+t$. In both regions $\underline{\delta} < \kappa$, with $\underline{\delta} = \kappa$ at $t = 0$. Equivalently, writing $\Gamma \equiv \pi(p,c) - \pi(p, c+t) \geq 0$ for the cost advantage a member enjoys at home over its partner's market, both branches collapse to the single expression

$$\underline{\delta} = 1 - \frac{1}{2n}\left(1 + \frac{\Gamma}{\Omega}\right), \tag{9}$$

since $\Gamma = tD(p)$ on the export-deviation region and $\Gamma = \pi(p,c)$ on the home-deviation region.

b) The minimum discount factor $\underline{\delta}$ depends on p, t, and π_{ROW} as follows:

(i) On the export-deviation region ($p > c+t$): $\underline{\delta}_p = \frac{t}{2n} \cdot \frac{2D(p)^2 - \pi_{ROW} D'(p)}{\Omega^2} > 0$. This inequality holds for any *demand function and does not invoke (A2). Moreover, $\underline{\delta}_t < 0$ and $\partial\underline{\delta}/\partial\pi_{ROW} = \frac{tD(p)}{2n\,\Omega^2} > 0$.*

(ii) On the home-deviation region ($p \in (c, c+t]$): $\underline{\delta}$ is independent of t and $\underline{\delta}_p = -\frac{\pi_{ROW}}{2n} \cdot \frac{\pi_p(p,c)}{\Omega^2} \leq 0$, with strict inequality for $\pi_{ROW} > 0$.

Hence $\underline{\delta}$ is U-shaped in p, attaining its minimum at the junction $p = c+t$.

c) If $\pi_{ROW} = 0$, then $\underline{\delta}$ is independent of demand: $\underline{\delta} = \frac{(2-\frac{1}{n})(p-c)-t}{2(p-c)-t}$ on the export-deviation region and $\underline{\delta} = 1 - \frac{1}{n}$ on the home-deviation region.

d) Because neither Γ nor Ω depends on n, (9) implies $\partial\underline{\delta}/\partial n = \frac{1}{2n^2}\left(1 + \frac{\Gamma}{\Omega}\right) > 0$, with $\underline{\delta} \to 1$ as $n \to \infty$: every agreement requires more patience when more firms reside in each host. No sign in part (b) depends on n. Consequently the efficient agreement (p^*, π^*_{ROW}) of

[38] Profit levels below π^m_{ROW} could also be implemented by prices above p^m_{ROW}. We adopt the natural selection of the lower price, which weakly benefits *ROW* consumers and involves no loss of generality for cartel members.

Proposition 7 is weakly decreasing in n at every t, and collusion is sustainable at all only if $n < 1/(1-\delta)$.

Lemma 4 overturns the benchmark point by point. Part (b) states that the minimum discount factor *falls* with trade costs: with the punishment payoff pinned at zero, only the deviation channel operates, and higher trade costs shrink the gain from invading the partner's market. Fragmentation is therefore unambiguously pro-collusive under domestic competition—the reversal of Proposition 1(a). Part (b) also states that access to *ROW* now *tightens* the ICC—the reversal of Proposition 1($b.ii$). The intuition rests on a comparison of standalone patience requirements: sustaining collusion in *ROW* alone requires $\delta \geq \kappa$, whereas the pooled home-and-partner markets require strictly less than κ whenever $t > 0$, because trade costs make the foreign deviation less tempting than the domestic one. Pooling in the tighter market raises the blend. Part ($b.ii$) reveals a further novelty: on the home-deviation region, raising the collusive price *relaxes* the ICC, because it shifts the composition of collusive profits toward the slacker home markets. Finally, the closed forms in part (a) imply $\underline{\delta} < \kappa$ for every price and every $t > 0$: under domestic competition, no collusive agreement—including maximal collusion—ever requires more patience than the classic threshold κ for frictionless Bertrand competition among the $2n$ firms.

Part (d) separates two things that are easily conflated. Collusion does get harder as the number of firms per host rises—by (9) the patience requirement increases in n and approaches 1, and by Proposition 7 the efficient agreement weakens accordingly, until collusion becomes unsustainable altogether once $n \geq 1/(1-\delta)$. But that is a statement about *levels*. No comparative static changes sign anywhere on $n \geq 2$: the trade-cost and *ROW*-profitability effects are the same for two firms per host as for two hundred. The reversal relative to Sections 3 and 4 is therefore not a gradient in the number of firms but a discontinuity between $n = 1$ and $n = 2$, and (9) locates it precisely. Its source is the punishment payoff, which equals the limit-pricing profit $tD(c+t)$ when a single firm resides in each host and is exactly zero for every $n \geq 2$.

Remark 4 *(Market structure as an observable index) The mechanism behind our central result is thus the* trade-cost sensitivity of punishment-phase profits*: fragmentation disciplines a cartel only when reversion to competition leaves its members a profit that trade costs erode. Domestic market structure matters because it settles that sensitivity—trade costs shelter a cartel member after a breakdown only if no domestic rival competes the price down to marginal cost. We nonetheless index our results by market structure rather than by punishment sensitivity, and deliberately so: the former is observable and the latter is not. A competition authority can count the firms resident in a cartel host; it cannot observe the profit a deviator would earn along a path that is never played. The same consideration* *reinforces the reading of Appendix C, where the discontinuity disappears under maximal punishments: what an outside observer can condition on is the market structure, not the penal code.*

5.2 Efficient Agreements

The cartel solves $\max_{p,p_{ROW}} \Pi^a$—equivalently, $\max_{p,\pi_{ROW}} \Pi^a$—subject to $\Phi \geq 0$, $p \in [c, p^m]$ and $\pi_{ROW} \in [0, \pi^m_{ROW}]$.[39] Define $\underline{\delta}^m(t) \equiv \underline{\delta}(p^m, t, \pi^m_{ROW})$ and $\underline{\delta}^J(t) \equiv \underline{\delta}(c+t, t, \pi^m_{ROW})$: the minimum discount factors required to sustain, respectively, the monopoly price and the junction price when the cartel fully monopolizes ROW. Both schedules decrease strictly in t, from the common value κ at $t = 0$ to a common limit at $t = \bar{t}$, with $1 - \frac{1}{n} < \underline{\delta}^J(t) < \underline{\delta}^m(t) < \kappa$ for $t \in (0, \bar{t})$. Fig. 4(a) depicts the two schedules and the regime partition they induce for a given δ.

Proposition 7 *(Efficient agreements under domestic competition) Suppose $n \geq 2$ and $t \in (0, \bar{t})$. The efficient agreement (p^*, π^*_{ROW}) is:*

a) If $\delta \geq \underline{\delta}^m(t)$: (p^m, π^m_{ROW}).

b) If $\delta \in [\underline{\delta}^J(t), \underline{\delta}^m(t))$: (p^c, π^m_{ROW}), where $p^c \in [c+t, p^m)$ solves $\underline{\delta}(p^c, t, \pi^m_{ROW}) = \delta$ and satisfies

$$\partial p^c/\partial t > 0, \;\; \partial p^c/\partial \delta > 0, \;\; \text{and} \;\; \partial p^c/\partial \pi^m_{ROW} = -\frac{D(p^c)}{2D(p^c)^2 - \pi^m_{ROW} D'(p^c)} < 0.$$

c) If $\delta \in (1 - \frac{1}{n}, \underline{\delta}^J(t))$: $(c+t, \hat{\pi}_{ROW})$, where $\hat{\pi}_{ROW} \in (0, \pi^m_{ROW})$: the cartel strictly restrains its ROW profits below the monopoly level, and $\hat{\pi}_{ROW}$ is independent of π^m_{ROW}, hence of external trade costs τ and of demand in ROW. $\hat{\pi}_{ROW}$ is strictly increasing in t and, provided $\delta \geq \underline{\delta}^J(\bar{t})$ (so that $t_J \in (0, \bar{t}\,]$ exists), reaches π^m_{ROW} as $t \to t_J$, where t_J solves $\underline{\delta}^J(t_J) = \delta$; it is implemented by a unique ROW price $\hat{p}_{ROW} \in (c+\tau, p^m_{ROW})$.

d) If $\delta \leq 1 - \frac{1}{n}$: $(c, 0)$; no collusion is sustainable.

Moreover, $p^*(t)$ and $\pi^*_{ROW}(t)$ are continuous and weakly increasing in t throughout $[0, \bar{t}]$.

In words: under domestic competition the efficient cartel agreement strengthens monotonically with fragmentation—no collusion in an integrated market, constrained collusion

[39] As noted earlier, we take the profit level π_{ROW}, rather than the price p_{ROW}, as the cartel's choice variable in ROW. This is without loss of generality and offers three advantages. First, p_{ROW} enters the cartel's objective and the ICC only through the profit it generates, so π_{ROW} is the payoff-relevant statistic; by Lemma A1, any target $\pi_{ROW} \in [0, \pi^m_{ROW}]$ is implemented by a unique price on $[c+\tau, p^m_{ROW}]$, from which the efficient p^*_{ROW} is recovered. (Implementations on the decreasing branch of the ROW profit function leave all cartel payoffs unchanged while reducing ROW consumer surplus, so restricting attention to $p_{ROW} \leq p^m_{ROW}$ is innocuous.) Second, the substitution renders the ICC linear in the choice variable, which delivers the transparent ray-and-junction geometry exploited in the proof of Proposition 7. Third, since π_{ROW} depends negatively on external trade costs τ, this formulation subsumes the importance of these costs.

at moderate trade costs, monopoly pricing beyond—and an impatient cartel protects the agreement by deliberately earning less in ROW.

Proposition 7 contains two prominent messages. The first is the monotonicity of collusion in trade costs. At $t = 0$ the cartel is simply a collection of $2n$ Bertrand competitors and no collusion is sustainable for $\delta < \kappa$ ($\equiv 1 - \frac{1}{2n}$); as t rises, collusion sets in and the efficient price climbs, first, along $c + t$, then along p^c, and finally at p^m (Fig. 4(b)). There is no non-monotonicity of the kind found in Propositions 2 and 3: under domestic competition, fragmentation only strengthens the cartel. The second message is part (c): when the cartel is impatient, it deliberately *restrains* its extraction in ROW. The logic follows from (8): ROW profits are deviation bait for the cartel's $2n$ members, so an impatient cartel moderates its own profits abroad—for example, by pricing below the π^m_{ROW} monopoly level—in order to protect collusion at home. The binding ICC at the junction price delivers the closed form

$$\hat{\pi}_{ROW} = k\, tD\,(c+t)\,, \qquad k \equiv \frac{\delta - 1 + \frac{1}{n}}{1 - \delta - \frac{1}{2n}} > 0,$$

where both the numerator and the denominator of k are positive in this regime (since $1 - \frac{1}{n} < \delta < \underline{\delta}^J(t) < \kappa$): the optimal restraint is proportional to the home limit-pricing profit $tD\,(c+t)$, with a factor that rises with the cartel's patience. Fig. 4(c) traces the resulting profile of $\pi^*_{ROW}(t)$, which rises from 0 at $t = 0$ to π^m_{ROW} at t_J and remains there for all larger trade costs. While this *characterization* of self-restraint in ROW is, to our knowledge, new, its direction was anticipated informally in prior work under maximal punishments (see the Introduction). Higher ROW profitability also depresses the constrained home price (part (b)), the mirror image of the benchmark result in Proposition 3($b.iii$).

Remark 5 *(External trade costs and the neutrality of third-market liberalization) Because profits in ROW depend on external trade costs τ, letting τ vary is the domestic-competition counterpart of the exercise in Proposition 5. Yet the two market structures could hardly differ more. Under domestic monopoly the cartel always extracts the monopoly rent abroad (i.e., $\pi^*_{ROW} = \pi^m_{ROW}(\tau)$), so external liberalization passes straight through to the home market: it increases π^*_{ROW}, relaxes the ICC, raises the collusive price, and can immiserize the hosts (Propositions 3($b.iii$) and 5). Under domestic competition the effect depends on the regime. In the regime of Proposition 7(b) the sign reverses: a higher π^m_{ROW} tightens the ICC and lowers the constrained home price, so liberalization abroad is anti-collusive and welfare-enhancing for hosts. In the regime of Proposition 7(c) it is neutralized. Since the optimal restraint $\hat{\pi}_{ROW} = k\, tD(c+t)$ contains no ROW object—neither τ, nor demand traits there, nor π^m_{ROW}—the cartel's foreign profit is pinned down entirely by its domestic incentive problem. While $\hat{\pi}_{ROW} < \pi^m_{ROW}$, a fall in τ therefore leaves the cartel's ROW*

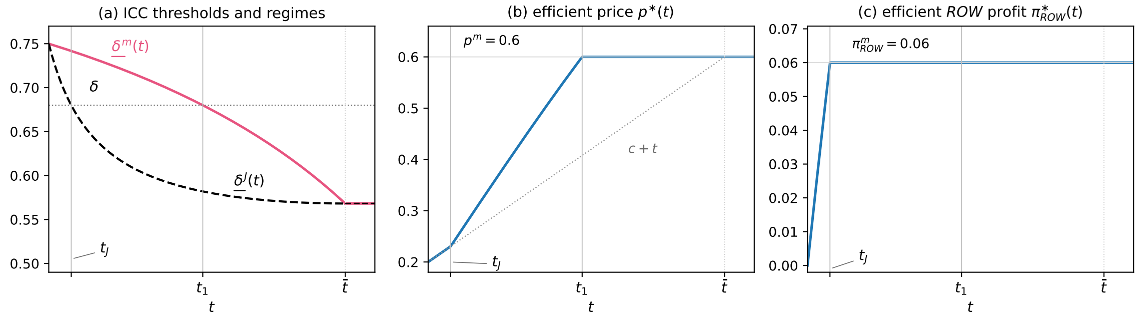

Figure 4: Efficient agreements under domestic competition: (a) the schedules $\underline{\delta}^m(t)$ and $\underline{\delta}^J(t)$ and the regimes of Proposition 7; (b) the efficient home price $p^*(t)$; (c) the efficient ROW profit $\pi^*_{ROW}(t)$. Linear-demand illustration with $D(p) = 1 - p$, $c = 0.2$, $n = 2$, $\pi^m_{ROW} = 0.06$, $\delta = 0.68$. Color coding follows Figs. 1–2: pink for the monopoly schedule, black for threshold loci, blue for efficient objects. Trade costs extend marginally beyond $\bar{t} = 0.4$: the two schedules in panel (a) merge at $\bar{t}$ and remain constant at $\underline{\delta}^J(\bar{t})$ beyond it—the home-deviation value of Lemma 4, which is independent of t—while $p^* = p^m = 0.6$ and $\pi^*_{ROW} = \pi^m_{ROW}$ beyond $\bar{t}$.

profit, its home price, and host welfare all unchanged: the cartel absorbs the improvement in foreign market access by restraining itself further, and the entire gain accrues to ROW consumers through the lower price that implements the same $\hat{\pi}_{ROW}$ at a lower delivered cost. Global welfare rises, but none of the gain is transmitted to the cartel or to its hosts. Note finally that a fall in τ raises $\underline{\delta}^J(t)$ and hence t_J, so liberalization abroad enlarges *the set of trade costs over which the cartel restrains itself, thus rendering the neutrality regime more likely, not less.*

The first message deserves a sharper statement, which follows directly from Proposition 7 and the fact that $\underline{\delta} = \kappa$ at $t = 0$:

Corollary 1 *(Fragmentation ignites collusion) Suppose $n \geq 2$ and $\delta \in (1 - \frac{1}{n}, \kappa)$. At $t = 0$ no collusion is sustainable, whereas for every $t \in (0, \bar{t})$ the efficient agreement involves strictly positive markups at home and strictly positive collusive profits in ROW.*

5.3 Welfare

Host welfare aggregates over the n resident firms: $V = v(p) + \bar{Y} + \pi(p, c) + \frac{1}{2}\pi_{ROW}$, so that (7) continues to apply. Global welfare adds ROW's surplus: $W = 2v(p) + 2\pi(p, c) + v_{ROW}(p_{ROW}) + \pi_{ROW} +$ constants, where $v'_{ROW} = -D_{ROW}$.

Proposition 8 *(Fragmentation and welfare under domestic competition) Suppose $n \geq 2$. Then:*

a) *Global welfare W^* is weakly decreasing in t on $[0, \bar{t}]$, and strictly decreasing on the regimes of parts (b) and (c) of Proposition 7.*

b) *In the regime of Proposition $7(c)$, host welfare rises with mild fragmentation: $dV^*/dt > 0$ for t sufficiently small.*

c) *The gain in part (b) reflects rent extraction from ROW: the rent hosts obtain from a higher ROW price is strictly smaller than the consumer surplus ROW loses. The alignment between host and global welfare that obtains under domestic monopoly breaks down under domestic competition.*

Proposition 8 disciplines the interpretation of any apparent "silver lining" of fragmentation. Under domestic competition, both equilibrium prices are weakly increasing in trade costs, so fragmentation unambiguously reduces global welfare. Cartel hosts may nonetheless gain from mild fragmentation—but not because trade barriers discipline the cartel. (In the expression for dV^*/dt in the proof, the hosts' share of the rising ROW rent, $\frac{k}{2}D\,(c+t)$, dominates the domestic distortion term when t is small.) On the contrary: small trade barriers *ignite* collusion where none was sustainable, and the hosts' gain is a beggar-thy-neighbor transfer extracted from ROW consumers, purchased at the price of a distortion at home and a larger loss abroad. Under domestic monopoly, by contrast, the cartel extracts monopoly rents from ROW at all times, so host and global welfare move together (Section 4); it is precisely the endogeneity of π^*_{ROW} under domestic competition that severs this link.

6 Concluding Remarks

Motivated by the ongoing fragmentation of the world economy, we asked whether rising trade barriers can discipline international cartels and improve welfare. To answer this question, we constructed a symmetric, homogeneous-goods model in which cartel members located in two host countries interact repeatedly in multiple markets—their own, each other's, and *ROW*—and sustain collusion by pooling incentive constraints across all of them. Demand functions were left general, restricted only by the requirement that they not be excessively convex (Assumption (A2)), a condition that delivers a unique monopoly price, strict concavity of profits below it, and, in the benchmark, the monotonicity of the minimum discount factor in the collusive price. Because (A2) restricts a primitive rather than the profit function, it admits convex demands—including the constant-elasticity form that the profit-concavity assumptions common in this literature exclude—while still delivering these properties.

Our central finding gives the title's question a sharp answer: domestic market structure determines whether fragmentation disciplines international cartels or entrenches them. Under domestic monopoly, the punishment phase involves limit pricing whose severity erodes as trade costs rise. Fragmentation can then destabilize collusion: efficient collusive prices are non-monotone in trade costs, host welfare is hump-shaped, intra-regional trade liberalization is welfare-reducing when initial trade costs are moderate, and profitable access to ROW reinforces the cartel—to the point that improvements in ROW profit opportunities can immiserize cartel hosts—a conclusion we establish for general demand and show to hold, for weakly convex demand such as the constant-elasticity case, throughout the constrained region. Under domestic competition, the punishment channel vanishes and every one of these conclusions gets reversed. Fragmentation is unambiguously pro-collusive; no agreement ever requires more patience than the classic threshold for frictionless Bertrand competition among all $2n$ cartel members; access to ROW undermines the cartel, inducing impatient cartels to deliberately restrain their own profits abroad—so much so that, for an impatient cartel, liberalization in third markets is fully absorbed by that self-restraint and leaves home prices and host welfare untouched, its entire benefit accruing to third-market consumers; and although hosts may gain from mild fragmentation, the gain is rent extraction from third markets, while global welfare falls monotonically as barriers rise.

These results carry two lessons for policy. First, claims that fragmentation may have a competitive silver lining should be evaluated jointly with the antitrust conditions prevailing inside the cartel's home markets: the silver lining exists only where host markets are domestically monopolized. Second, because collusive agreements in prices never involve cross-hauling, fragmentation affects collusion and welfare without leaving a trace in trade flows: as we emphasized in Section 4 (Remark 3), stable or declining import volumes are uninformative about cross-border collusion, and price-based screens—conditioned on domestic market structure—are the appropriate detection instruments.

Two remarks on the scope of these conclusions. First, the reversal is not an artifact of the punishment convention; it is a statement about it. Because the minimum discount factor is increasing in the punishment payoff, the set of sustainable agreements under *any* credible punishment value between the limit-pricing profit $tD(c+t)$ and zero is nested between the two cases we characterize—the benchmark of Sections 3 and 4 and Section 5 read at $n = 1$—so the model with optimal penal codes is bracketed by results already in hand (Appendix C). What the two conventions do is relocate the same mechanism: fragmentation disciplines a cartel only when reversion to competition leaves its members a profit that trade costs erode. Second, our framework is deliberately stylized: firms are symmetric, the product is homogeneous, entry is precluded, and punishment takes the grim-trigger form. These choices isolate the punishment and deviation channels through which trade costs operate. We see at least three directions in which the analysis could be extended. First, exporting firms often incur per-period fixed costs in addition to variable trade costs (Roberts and Tybout, 1997; Melitz, 2003); how such costs affect collusive outcomes and welfare remains open. Second, multinational firms may serve foreign markets through foreign direct investment rather than exports, trading fixed operating costs for variable trade costs (Helpman et al., 2004); it would be of interest to know how this option interacts with cartel discipline in our setting. Third, asymmetries in marginal production costs across cartel members would introduce limit-pricing considerations within the cartel itself. We plan to explore these issues in future research.

References

Abreu, D., 1986. "Extremal Equilibria of Oligopolistic Supergames," *Journal of Economic Theory*, vol. 39(1), pp. 191-225.

Abreu, D., 1988. "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, vol. 56(2), pp. 383-396.

Agnosteva, D., Syropoulos, C., and Yotov, V.Y., 2024. "Preferential Trade Liberalization with Endogenous Cartel Discipline: Implications for Trade and Welfare," *Canadian Journal of Economics*, vol. 57(4), pp. 1109-1136.

Aiyar, S., Ilyina, A., and others, 2023. "Geoeconomic Fragmentation and the Future of Multilateralism," Staff Discussion Note SDN/2023/001. International Monetary Fund, Washington, DC.

Akinbosoye, O., Bond, E.W. and Syropoulos, C., 2012. "On the Stability of Multimarket Collusion in Price-Setting Supergames," *International Journal of Industrial Organization*, vol. 30(2), pp. 253-264.

Ashournia, D., Hansen, P.S., and Hansen, J.W., 2013. "Trade Liberalization and the Degree of Competition in International Duopoly," *Review of International Economics*, vol. 21(5), pp. 1048-1059.

Bernheim, D.B. and Whinston, M.D., 1990. "Multimarket Contact and Collusive Behavior," *RAND Journal of Economics*, vol. 21(1), pp. 1-26.

Bhattacharjea, A. and Sinha, U.B., 2015. "Multi-market Collusion with Territorial Allocation," *International Journal of Industrial Organization*, vol. 41, pp. 42-50.

Bond, E.W. and Syropoulos, C., 2008. "Trade Costs and Multimarket Collusion," *RAND Journal of Economics*, vol. 39(4), pp. 1080-1104.

Bos, I. and Harrington, J.E., 2010. "Endogenous Cartel Formation with Heterogeneous Firms," *RAND Journal of Economics*, vol. 41(1), pp. 92-117.

Brander, J.A. and Krugman, P., 1983. "A Model of Reciprocal Dumping," *Journal of International Economics*, vol. 5, pp. 313-323.

Connor, J.M., 2010. "Recidivism Revealed: Private International Cartels 1990–2009," *CPI Journal, Competition Policy International*, vol. 6(2), pp. 101-127.

Davidson, C., 1984. "Cartel Stability and Tariff Policy," *Journal of International Economics*, vol. 17, pp. 219-237.

Deltas, G., Salvo, A. and Vasconcelos, H., 2012. "Consumer-Surplus-Enhancing Collusion and Trade," *RAND Journal of Economics*, vol. 43(2), pp. 315-328.

European Commission, 2005. Commission Decision 2005/566/EC of 9 December 2004 relating to a proceeding under Article 81 of the EC Treaty and Article 53 of the EEA Agreement (Case COMP/E-2/37.533—Choline Chloride), *Official Journal of the European Union*, L 190, 22.7.2005, pp. 22-26.

Fajgelbaum, P.D., Goldberg, P.K., Kennedy, P.J., and Khandelwal, A.K., 2020. "The Return to Protectionism," *Quarterly Journal of Economics*, vol. 135(1), pp. 1-55.

Harrington, J.E., 2006. "How Do Cartels Operate?" *Foundations and Trends in Microeconomics*, vol. 2(1), pp. 1-105.

Harrington, J.E., 1991. "The Determination of Price and Output Quotas in a Heterogeneous Cartel," *International Economic Review*, vol. 32(4), pp. 767-792.

Head, K. and Spencer, B.J., 2017. "Oligopoly in International Trade: Rise, Fall and Resurgence," *Canadian Journal of Economics*, vol. 50(5), pp. 1414-1444.

Helpman, E., Melitz, M.J. and Yeaple, S.R., 2004. "Export Versus FDI with Heterogeneous Firms," *American Economic Review*, vol. 94(1), pp. 300-316.

Levenstein, M.C. and Suslow, V.Y., 2011. "Breaking Up Is Hard to Do: Determinants of Cartel Duration," *Journal of Law and Economics*, vol. 54(2), pp. 455-492.

Lommerud, K.E. and Sørgard, L., 2001. "Trade Liberalization and Cartel Stability," *Review of International Economics*, vol. 9, pp. 343-355.

Melitz, M.J., 2018. "Competitive Effects of Trade: Theory and Measurement," *Review of World Economics*, vol. 154(1), pp. 1-13.

Melitz, M.J., 2003. "The Impact of Trade on Aggregate Industry Productivity and Intra-Industry Reallocations," *Econometrica*, vol. 71(6), pp. 1695-725.

Neary, J.P., 2003. "Presidential Address: Globalization and Market Structure," *Journal of the European Economic Association*, vol. 1(2-3), pp. 245-271.

Pinto, B., 1986. "Repeated Games and the 'Reciprocal Dumping' Model of Trade," *Journal of International Economics*, vol. 20, pp. 357-366.

Roberts, M.J. and Tybout, J.R., 1997. "The Decision to Export in Colombia: An Empirical Model of Entry with Sunk Costs," *American Economic Review*, vol. 87(4), pp. 545-564.

Schröder, P.J.H., 2007. "Cartel Stability and Economic Integration," *Review of International Economics*, vol. 15(2), pp. 313-320.

Syropoulos, C., 1992. "Quantitative Restrictions and Tariffs with Endogenous Firm Behavior," *European Economic Review*, vol. 36, pp. 1627-1646.

United States Trade Representative, 2026. "Ambassador Greer Issues Statement on the USMCA Joint Review," press release, July 1, 2026. `https://ustr.gov`.

Appendix A: Proofs

This appendix proves the results not established in the text, with five items deferred to the Supplementary Appendix for online publication: the complete proofs of Lemma 3, Proposition 1, and Proposition 6 (Sections S1–S3), of which the first two are sketched below; Appendix B, which proves part (a) of Proposition 5 and develops the role of demand curvature; and Appendix C, which discusses the punishment convention.

Lemma A1: Our assumptions on demand functions together with (A2), imply that firm i's profit function $\pi_{ij} = \pi(p, c_{ij})$ is: (a) strictly quasi-concave in price $p \in (c_{ij}, \tilde{p}_j)$ (where $\tilde{p}_j > \max_i\{c_{ij}\}$) and, therefore, admits a unique global maximum $p_{ij}^m = p^m(c_{ij})$; (b) is strictly concave in p for all $p \in (c_{ij}, p_{ij}^m)$.

Proof: Differentiation of $\pi(p, c_{ij})$ with respect to price gives $\pi_p = D(p) + (p - c_{ij})D'(p)$, which implies $\pi_p|_{p=c_{ij}} > 0$. But $\pi|_{p=c_{ij}} = 0 = \pi|_{p=\tilde{p}_j}$ and, as just noted, $\pi_p > 0$ in the neighborhood of $p = c_{ij}$. It follows that there exists at least one price $p_{ij}^m \in (c_{ij}, \tilde{p}_j)$ that solves $\pi_p(\cdot) = 0$, thereby ensuring $\pi(\cdot)$ attains a maximum. It is unclear at this point whether p_{ij}^m is unique.

Next, differentiate π_p with respect to price p to obtain $\pi_{pp} = 2D'(p) + (p - c_{ij})D''(p)$ and use the first-order condition $\pi_p = 0$ for an interior solution to write $p - c_{ij} = -D(p)/D'(p)$. Substituting this expression into π_{pp} and rearranging terms delivers $\pi_{pp}|_{p=p_{ij}^m} = D'(\cdot)\eta(\cdot)$, which is negative due to $D'(\cdot) < 0$ and (A2). Hence every interior critical point of π is a strict local maximum; since two such maxima would have to be separated by an interior critical point that is not a local maximum, the critical point is unique. Therefore π is strictly quasi-concave in p and p_{ij}^m is unique. Clearly, then, $\pi_p > 0$ for all prices $p \in [c_{ij}, p_{ij}^m)$.

We will now prove that $\pi(p, c_{ij})$ is strictly concave in price for all $p \in [c_{ij}, p_{ij}^m]$. For concave demand functions the statement is immediate, since then $\pi_{pp} = 2D'(p) + (p - c_{ij})D''(p) < 0$ for all $p > c_{ij}$; it remains to show that it also holds when $D''(p) > 0$ (or, equivalently, when $\gamma(p) > 0$) under (A2). One can verify that $\pi_{pp} = D'(p)\,[\eta(p) + \pi_p(\cdot)\gamma(p)/D(p)]$. Since $\eta(p) > 0$ by (A2), $\pi_p \geq 0$ for $p \in [c_{ij}, p_{ij}^m]$, and $\gamma(p) > 0$, we will have $\pi_{pp} < 0$; thus $\pi(p, c_{ij})$ is strictly concave in p for the range of prices considered. ||

Proof of Lemma 3 (sketch): Differentiation of $\underline{\delta}$ in (6) gives

$$\underline{\delta}_p = \frac{tD(p)}{\Omega^2}\left[-\frac{1}{2}\pi^*_{ROW}\frac{D'(p)}{D(p)} + D(p^n)\Lambda(p,t)\right], \qquad \Lambda(p,t) \equiv \left[\frac{D(p)}{D(p^n)} - 1\right] - (p-p^n)\frac{D'(p)}{D(p)}.$$

The first term in brackets is non-negative, so the sign of $\underline{\delta}_p$ turns on that of Λ. Since $\Lambda \to 0$ as $p \to p^n$, a violation would require $\Lambda_p \leq 0$ at some price at which $\Lambda \leq 0$; but $\Lambda \leq 0$ implies $\Lambda_p \geq (p - p^n)\,[D'(p)/D(p)]^2\,\eta(p) > 0$ by (A2)—a contradiction. Hence $\Lambda > 0$ and $\underline{\delta}_p > 0$. The limits in parts (a) and (b) follow by L'Hôpital's rule. Section S1 of the Supplementary Appendix gives the complete argument. ||

Proof of Proposition 1 (sketch): Differentiating $\underline{\delta}$ with respect to t, and recalling $p^n = c + t$, gives $\underline{\delta}_t = \Omega^{-2}\left\{\frac{1}{2}\pi^*_{ROW}\,[\pi_p(p^n, c) - D(p)] + D(p)\,\Theta(p,t)\right\}$, where $\Theta(p,t) \equiv \pi(p^n, c) + (p - p^n)\pi_p(p^n, c) - \pi(p, c) > 0$, the sign following from the strict concavity of $\pi(\cdot, c)$ below p^m (Lemma A1). If $\pi^*_{ROW} = 0$ the first term vanishes and $\underline{\delta}_t > 0$, which is part (a); the limits in ($a.i$)–($a.iii$) follow from L'Hôpital's rule and from direct differentiation of $\underline{\delta}^x$, the latter invoking (A2) to sign $d\underline{\delta}^x/dp$. If $\pi^*_{ROW} > 0$ the first term is positive as $t \to 0$ and negative as $t \to t_x$, so an interior peak t_0 exists, and $\underline{\delta}_{tt}|_{t=t_0} < 0$ delivers strict quasi-concavity and uniqueness. Parts ($b.ii$)–($b.iv$) follow from $\partial\underline{\delta}/\partial\pi^*_{ROW} = -\frac{1}{2}\Psi/\Omega^2 \leq 0$, from $d\underline{\delta}^0/dp = \underline{\delta}_p > 0$, and from $\underline{\delta} < 1$. Section S2 of the Supplementary Appendix gives the complete argument. ||

Proof of Proposition 3: *Part (a).* This part holds true because $\underline{\delta}\,(p^m, t, \pi^*_{ROW}) \leq \underline{\delta}_{\max} \leq \delta$ for all $t \in [0, \bar{t}]$, so the ICC is satisfied for $p = p^m$ when $\delta \in [\underline{\delta}_{\max}, 1)$; optimality of p^m follows since Π^a is increasing in p.

Part (b). The existence and uniqueness of t_1 and t_2, as well as their properties described in part (i), follow from the properties of $\underline{\delta}^m$ described in part (b) of Proposition 1. Part

(ii) follows from the facts that $\partial p^c/\partial t = -\underline{\delta}_t/\underline{\delta}_p$, where $\underline{\delta}_p > 0$ from Lemma 3 and $\underline{\delta}_t \gtreqless 0$ as $t \lesseqgtr t_0$ for $t \in (t_1, t_2)$. Here $t_0 = t_0\,(p_0, \pi^*_{ROW})$ with $\underline{\delta}^0\,(p_0) = \delta$: along the binding path, $\underline{\delta}_t$ changes sign only where the path meets the locus $\underline{\delta}^0$, and this tangency point is unique because $\partial\underline{\delta}^0/\partial p > 0$ (Proposition 1$(b.iii)$) while the two branches of the level set $\underline{\delta} = \delta$ merge exactly there. The inequalities in part (iii) follow from the facts that $\partial p^c/\partial\delta = 1/\underline{\delta}_p > 0$ while $\partial p^c/\partial\pi^*_{ROW} = -\underline{\delta}_{\pi^*_{ROW}}/\underline{\delta}_p$ with $\underline{\delta}_{\pi^*_{ROW}} < 0$ from part $(b.ii)$ of Proposition 1. ||

Proof of Proposition 6: Proposition 6 specializes Proposition 5 to linear demand; because its content is the closed forms rather than a new argument, the proof—together with the closed-form expressions for the peak of the constrained path to which the text refers—is given in Section S3 of the Supplementary Appendix. ||

Proof of Lemma 4: Part (a): Since $\Pi^n = 0$, the definition of $\underline{\delta}$ gives $\underline{\delta} = \left(\Pi^d - \Pi^a\right)/\Pi^d$, i.e., $\Pi^a = (1-\underline{\delta})\Pi^d$; substituting into $\Phi = \Pi^a - (1-\delta)\Pi^d$ yields $\Phi = (\delta - \underline{\delta})\Pi^d$. Direct computation of $\Pi^d - \Pi^a$ gives $\underline{\delta} = \left[(1-\frac{1}{n})\pi(p,c) + \pi(p,c+t) + \kappa\pi_{ROW}\right]/\Omega$ on the export-deviation region (with $\pi(p,c+t)$ replaced by 0 on the home-deviation region). Subtracting $\kappa\Omega$ from the numerator and noting $\pi(p,c) - \pi(p,c+t) = tD(p)$ delivers $\underline{\delta} = \kappa - \frac{tD(p)}{2n\Omega}$ on the export-deviation region; on the home-deviation region the same subtraction delivers $\underline{\delta} = \kappa - \frac{\pi(p,c)}{2n\Omega}$ directly. The two expressions coincide at $p = c+t$, where $\pi(p,c) = tD(p)$.

Part $(b.i)$: Differentiating $\underline{\delta}$ on the export-deviation region and simplifying (the computation is mechanical) yields the stated closed form for $\underline{\delta}_p$, which is positive because $D > 0$ and $D' < 0$; no restriction on the curvature of D is used. Similarly, $\underline{\delta}_t = -\frac{D(p)}{n}\left[\pi(p,c) + \frac{1}{2}\pi_{ROW}\right]/\Omega^2 < 0$ and $\partial\underline{\delta}/\partial\pi_{ROW} = \frac{tD(p)}{2n\Omega^2} > 0$.

Part $(b.ii)$: On the home-deviation region, $\underline{\delta}$ reduces to

$$\underline{\delta} = \left[\left(1-\tfrac{1}{n}\right)\pi(p,c) + \kappa\pi_{ROW}\right] \Big/ \left[\pi(p,c) + \pi_{ROW}\right],$$

which contains no t; differentiating with respect to p gives $\underline{\delta}_p = -\frac{\pi_{ROW}}{2n}\pi_p(p,c)/\Omega^2$, where $\pi_p(p,c) > 0$ for $p < p^m$ by Lemma A1 (this is where Assumption (A2) enters). Since $\underline{\delta}$ is decreasing in p on the home-deviation region and increasing on the export-deviation region, and the two branches coincide at $p = c+t$, $\underline{\delta}$ is U-shaped with minimum at the junction.

Part (c): Set $\pi_{ROW} = 0$; the common factor $D(p)$ cancels from every term of $\underline{\delta}$, leaving the stated demand-free expressions.

Part (d): For the collapsed form (9), note that $\kappa - \frac{G}{2n\Omega} = 1 - \frac{1}{2n} - \frac{G}{2n\Omega} = 1 - \frac{1}{2n}\left(1 + \frac{G}{\Omega}\right)$, and that $\Gamma = \pi(p,c) - \pi(p,c+t)$ equals $tD(p)$ when $p > c+t$ and $\pi(p,c)$ when $p \le c+t$ (where $\pi(p,c+t) = 0$), which reproduces both branches of part (a). Since $\Gamma \ge 0$ and $\Omega > 0$ are functions of (p, t, π_{ROW}) alone, differentiating (9) with respect to n gives the stated expression, which is positive; and $\underline{\delta} \to 1$ because $\frac{1}{2n}\left(1 + G/\Omega\right) \to 0$. That no sign in part

(b) depends on n follows because n enters each derivative there only through the positive scalar $\frac{1}{2n}$. For the efficient agreement, the three regime boundaries of Proposition 7—$\underline{\delta}^m(t)$, $\underline{\delta}^J(t)$, and $1-\frac{1}{n}$—are each strictly increasing in n, so for fixed (δ, t) an increase in n moves the cartel weakly down the ordered regimes $(a) \to (b) \to (c) \to (d)$; within regime (b), $\partial p^c/\partial n = -\left(\partial \underline{\delta}/\partial n\right)/\underline{\delta}_p < 0$ by part $(b.i)$, and within regime (c), k is strictly decreasing in n, so $\hat{\pi}_{ROW} = k\, tD(c+t)$ falls. Finally, regime (d) obtains exactly when $\delta \leq 1-\frac{1}{n}$, i.e. when $n \geq 1/\left(1-\delta\right)$. $\quad ||$

Proof of Proposition 7: We prove the proposition in five steps.

Step 1 (Reformulation). By Lemma 4(a), the feasible set is $\{(p, \pi_{ROW}) : \underline{\delta} \leq \delta\}$ and its boundary is the level curve $\underline{\delta} = \delta$. On this boundary $\Pi^a = (1-\delta)\Pi^d$; hence, whenever the ICC binds at the optimum, the optimum maximizes Π^d over the binding set. If instead the ICC is slack at the optimum, then—since Π^a is strictly increasing in both arguments—the optimum must be the unconstrained corner (p^m, π^m_{ROW}), which is feasible precisely when $\delta \geq \underline{\delta}^m\left(t\right)$. This proves part (a).

Step 2 (No interior tangency). An interior optimum on the binding set would require the gradients of Π^a and Π^d to be proportional (the Lagrange condition for $\max \Pi^a$ subject to $\Phi = 0$, given $\Phi = \Pi^a - (1-\delta)\Pi^d$); that is, $\frac{1}{n}\pi_p(p,c) \,/\, \left[\pi_p(p,c) + \pi_p(p,c+t)\right] = \frac{1}{2n}\,/1$, which reduces to $\pi_p(p,c) = \pi_p(p,c+t)$. But $\pi_p(p,c+t) - \pi_p(p,c) = -tD'(p) > 0$ for $t > 0$: a contradiction. The optimum therefore lies at the junction kink or at a box corner of the binding set.

Step 3 (Export-deviation branch). At any binding point with $p > c+t$, $\Phi_p = -\underline{\delta}_p \Pi^d < 0$ by Lemma 4(b). Parametrizing the branch by π_{ROW}, the implicit function theorem gives $dp/d\pi_{ROW} = -\left[\frac{1}{2n} - (1-\delta)\right]/\Phi_p < 0$ and

$$\left.\frac{d\Pi^d}{d\pi_{ROW}}\right|_{\Phi=0} = \frac{-tD'(p)/(2n)}{\underline{\delta}_p\, \Pi^d} > 0.$$

Hence Π^d rises along the branch as π_{ROW} increases: the cartel trades home price for *ROW* profit up to the corner $\pi_{ROW} = \pi^m_{ROW}$, provided the branch reaches it with $p \geq c+t$, i.e., provided $\delta \geq \underline{\delta}^J\left(t\right)$. This proves part (b); the comparative statics of p^c follow from the implicit function theorem and the derivatives in Lemma 4(b).

Step 4 (Home-deviation branch). For $\delta \in (1-\frac{1}{n}, \kappa)$, the binding set on the home-deviation region is the ray $\pi_{ROW} = k\,\pi(p,c)$ with $k = \left(\delta - 1 + \frac{1}{n}\right)/\left(1-\delta-\frac{1}{2n}\right) > 0$ (both numerator and denominator are positive for $\delta \in (1-\frac{1}{n}, \kappa)$), along which $\Pi^d = (1+k)\pi(p,c)$ is strictly increasing in p by Lemma A1. The branch therefore also points toward the junction $p = c+t$.

Step 5 (Assembly). Both branches of the binding set increase toward the junction

$(c+t, \hat{\pi}_{ROW})$, where $\hat{\pi}_{ROW} = k\, tD(c+t)$ solves $\Phi\,(c+t, \hat{\pi}_{ROW}) = 0$. If $\delta \geq \underline{\delta}^J\,(t)$, the export-deviation branch terminates at (p^c, π^m_{ROW}), which is then the optimum (part (b)); otherwise $\hat{\pi}_{ROW} < \pi^m_{ROW}$ and the junction is the optimum (part (c)). Since $d\hat{\pi}_{ROW}/dt = k\,\pi_p\,(c+t, c) > 0$ for $t < \bar{t}$ by Lemma A1, $\hat{\pi}_{ROW}$ rises with t and—when $t_J < \bar{t}$ exists, i.e., when $\delta \geq \underline{\delta}^J(\bar{t})$—equals π^m_{ROW} exactly at $t = t_J$, delivering the continuous pasting and the joint monotonicity of (p^*, π^*_{ROW}) in t; otherwise regime (c) prevails on all of $(0, \bar{t})$ and the same monotonicity holds. For $\delta \leq 1 - \frac{1}{n}$, every coefficient in Φ is non-positive (strictly negative for π_{ROW}), so no agreement with $p > c$ or $\pi_{ROW} > 0$ satisfies the ICC; this proves part (d). Finally, the properties of $\underline{\delta}^m$ and $\underline{\delta}^J$ asserted in the text follow from Lemma 4(b), the fact that $\underline{\delta}^J$ is decreasing in $tD(c+t)$ (which is itself increasing in t on $[0, \bar{t})$ by Lemma A1), and $\underline{\delta} \to \kappa$ as $t \to 0$. ||

Proof of Proposition 8: With $W = 2v(p) + 2\pi(p, c) + v_{ROW}(p_{ROW}) + \pi_{ROW} +$ constants, Roy's identity gives $dW/dp = 2\,(p-c)\,D'(p) < 0$ for $p > c$ and $dW/dp_{ROW} = (p_{ROW} - c - \tau)\,D'_{ROW}(p_{ROW}) < 0$ for $p_{ROW} > c + \tau$. By Proposition 7, $p^*(t)$ and $\pi^*_{ROW}(t)$—and hence $p^*_{ROW}(t)$—are weakly increasing in t in every regime, strictly in the regimes of parts (b) and (c). Part (a) follows. For part (b), in the regime of Proposition 7(c) host welfare is $V = v(c+t) + \left(1 + \frac{k}{2}\right) tD(c+t) + \bar{Y}$, with k as defined in the text; differentiation gives $dV^*/dt = \frac{k}{2} D\,(c+t) + \left(1 + \frac{k}{2}\right) tD'\,(c+t)$, whose limit as $t \to 0$ is $\frac{k}{2} D(c) > 0$. For part (c), the change in host income from a higher ROW price is $d\pi_{ROW}/dp_{ROW} = D_{ROW} + (p_{ROW} - c - \tau)\,D'_{ROW}$, which falls short of ROW's consumer surplus loss D_{ROW} by exactly $-\,(p_{ROW} - c - \tau)\,D'_{ROW} > 0$. Under domestic monopoly, $p^*_{ROW} = p^m_{ROW}$ is invariant, so v_{ROW} is constant and W moves with host welfare; under domestic competition, p^*_{ROW} varies with t and the alignment fails. ||

A Supplementary Appendix is provided for online publication. It contains five sections. Sections S1–S3 give the complete proofs of Lemma 3, Proposition 1, and Proposition 6, for the first two of which the sketches above record the key expressions and the decisive inequalities. Appendix B proves part (a) of Proposition 5 and develops the role of demand curvature (Lemma B1, Theorem B1, Corollary B1); Appendix C discusses the punishment convention and the bracketing argument of Section 2.

Supplementary Appendix

For Online Publication

Trade Fragmentation, International Cartels, and Welfare: How Does Domestic Market Structure Matter?

Delina E. Agnosteva Constantinos Syropoulos Yoto V. Yotov

July 31, 2026

This supplement contains the complete proofs of Lemma 3 (Section S1), Proposition 1 (Section S2), and Proposition 6 (Section S3), for the first two of which Appendix A of the main text gives sketches, together with Appendix B and Appendix C. Equations internal to this supplement are numbered (S.1), (S.2), and so on; every other equation, proposition, lemma and section number refers to the main text.

S1: Proof of Lemma 3

We first prove that $\underline{\delta}_p > 0$ for $p \in (p^n, p^m]$. Differentiation of $\underline{\delta}$ with respect to price along with the definitions of the per-period profits in (6) give

$$\underline{\delta}_p = \frac{tD(p)}{\Omega^2}\left[-\frac{1}{2}\pi^*_{ROW}\frac{D'(p)}{D(p)} + D(p^n)\Lambda(p,t)\right], \tag{S.1}$$

where

$$\Lambda = \Lambda(p,t) \equiv \left[\frac{D(p)}{D(p^n)} - 1\right] - (p-p^n)\frac{D'(p)}{D(p)}. \tag{S.2}$$

Since $D'(p) < 0$, the first term inside the brackets in the RHS of (S.1) is non-negative; therefore, everything else staying the same, a higher collusive profit in ROW makes more likely that $\underline{\delta}_p > 0$. But to sign $\underline{\delta}_p$ we also need to sign Λ which from (S.2) appears to be ambiguous for $p > p^n$ $(= c + t)$. We will show that Assumption (A2) ensures $\Lambda > 0$.

To start with note that $\lim_{p \to p^n} \Lambda = 0$. Now, contrary to our aim, suppose $\Lambda \leq 0$ for some $p \in (p^n, p^m]$. This supposition implies that, for Λ to remain equal to zero or turn negative for certain $p > p^n$, it must be the case that $\Lambda_p \leq 0$ at some p with $\Lambda(p) \leq 0$. We will argue that $\Lambda \leq 0$ always implies $\Lambda_p > 0$; therefore, our supposition that $\Lambda \leq 0$ cannot be correct.

Differentiation of Λ with respect to p gives

$$\Lambda_p = \left[\frac{D(p)}{D(p^n)} - 1\right]\frac{D'(p)}{D(p)} + (p-p^n)\left[\frac{D'(p)}{D(p)}\right]^2[1-\gamma(p)], \tag{S.3}$$

where, as described in Assumption (A2), $\gamma(p) \equiv \frac{D''(p)D(p)}{D'(p)^2}$. Our supposition that $\Lambda \leq 0$ implies $\frac{D(p)}{D(p^n)} - 1 \leq \frac{(p-p^n)D'(p)}{D(p)}$ or, equivalently, $\left[\frac{D(p)}{D(p^n)} - 1\right]\frac{D'(p)}{D(p)} \geq (p - p^n)\left[\frac{D'(p)}{D(p)}\right]^2$ for $p \in (p^n, p^m]$. Adding $(p - p^n)\left[\frac{D'(p)}{D(p)}\right]^2 [1 - \gamma(p)]$ to both sides of this inequality and noting that the LHS equals Λ_p enables us to rewrite it as

$$\begin{aligned}\Lambda_p &\geq (p - p^n)\left[\frac{D'(p)}{D(p)}\right]^2 + (p - p^n)\left[\frac{D'(p)}{D(p)}\right]^2 [1 - \gamma(p)] \\ &= (p - p^n)\left[\frac{D'(p)}{D(p)}\right]^2 [2 - \gamma(p)] \\ &= (p - p^n)\left[\frac{D'(p)}{D(p)}\right]^2 \eta(p) > 0.\end{aligned}$$

The above inequality follows from our focus on $p \in (p^n, p^m]$, (A2) and our supposition that $\Lambda \leq 0$. Since $\Lambda_p > 0$, it's impossible for Λ to turn non-positive. Therefore, $\Lambda > 0$ and $\underline{\delta}_p > 0$.

Part (a): Inspection of (6) reveals that, if $\pi^*_{ROW} = 0$, both the numerator and the denominator of $\underline{\delta}$ equal 0 when $p = p^n$, so the function is undefined at this value of p. Nonetheless, application of L'Hôpital's rule reveals that $\lim_{p \to p^n} \underline{\delta}$ is given by the expression in part (a), so it exists. It is also worth underlining the following ideas: First, $\lim_{p \to p^n} \underline{\delta} = \frac{1}{2}$ if $t = 0$ (or, equivalently, if $p^n = c$). Second, $\lim_{p \to p^n} \underline{\delta} = 1$ if $t = \bar{t}$ (or, equivalently, if $p^n = p^m$).[1]

Part (b): This part holds true because all the components of $\underline{\delta}$ in (6) vanish as $p \to p^n$, except for π^*_{ROW}. ||

S2: Proof of Proposition 1

Recall that the function $\underline{\delta}(\cdot)$ is strictly quasi-concave in t on T^x iff one of the following three conditions holds: (i) $\underline{\delta}(\cdot)$ is increasing in t; (ii) $\underline{\delta}(\cdot)$ is decreasing in t; (iii) there exists a t_0 such that $\underline{\delta}(\cdot)$ is increasing on $\{t \in T^x : t < t_0\}$ and decreasing on $\{t \in T^x : t > t_0\}$.

Differentiating $\underline{\delta}$ with respect to t, while keeping in mind that $p^n = c + t$, gives

$$\underline{\delta}_t = \frac{1}{\Omega^2}\left\{\frac{1}{2}\pi^*_{ROW}\left[\pi_p(p^n, c) - D(p)\right] + D(p)\,\Theta(p, t)\right\} \tag{S.4}$$

where

$$\Theta(p, t) = \pi(p^n, c) + (p - p^n)\,\pi_p(p^n, c) - \pi(p, c) > 0.$$

[1] This is so because $\pi_p(p^m, c) = D(p^m) + (p^m - c)D'(p^m) = 0$.

The positive sign of $\Theta(p,t)$ is due to the strict concavity of the profit function in $p \in (c_{ij}, p^m_{ij})$.[2] Equation (S.4) helps prove the remainder of the proposition.

Part (a). Since $\pi_{ROW} = 0$ in this part, from (S.4) we readily obtain $sign(\underline{\delta}_t) = sign(\Theta)$. Clearly, $\underline{\delta}_t > 0$ in this case. The pink curve in Fig. 1(a) depicts $\underline{\delta}$ as a function of t for $p = p^m$, and the blue and green curves for two prices $p < p^m$. Part (i) follows readily by noting that $\lim_{t\to 0} \pi(p, c+t) = \pi(p,c)$ and $\lim_{t\to 0} \pi(p^n, c) = 0$ in (6). To prove part (ii) note that both the numerator and denominator of $\underline{\delta}$ in (6) converge to 0 as $t \to t_x$; therefore, $\underline{\delta}$ is not defined at $t = t_x$. Nonetheless, the limit exists and can be obtained in a straightforward way by using L'Hôpital's rule.

By its definition, $\underline{\delta}^x(p)$ is the value of $\underline{\delta}$ as $t \to t_x$. The first two properties of $\underline{\delta}^x(p)$ in part (iii) follow upon inspection of its expression in this part.[3] The last part can be established by differentiating the expression for $\underline{\delta}^x$ in part $(a.ii)$ directly. Writing $G \equiv 2D(p) + (p-c)D'(p)$ (>0), we obtain $d\underline{\delta}^x/dp = -D'(p)\left[D(p) + (p-c)D'(p)(\gamma(p)-1)\right]/G^2$. If $\gamma \leq 1$, the bracketed term is at least $D(p) > 0$; if $\gamma \in (1,2)$—the remaining possibility under (A2)—the bracketed term strictly exceeds $D(p) + (p-c)D'(p) = \pi_p(p,c) \geq 0$ because $0 < \gamma - 1 < 1$ and $(p-c)D'(p) < 0$. In either case $d\underline{\delta}^x/dp > 0$. Schedule $\underline{\delta}^x$, which is captured by the black curve in Fig. 1(a), is an envelope of the maxima of the $\underline{\delta}$ curves shown there.[4]

Part (b). Focusing on $\pi^*_{ROW} > 0$, first note that $\lim_{t\to 0} \underline{\delta}_t > 0$ and $\lim_{t\to p-c} \underline{\delta}_t < 0$. The former inequality holds because $\lim_{t\to 0}\left[\pi_p(p^n,c) - D(p)\right] = D(c) - D(p) > 0$ (as D is decreasing in price) and $\Theta > 0$ for all $t \in T^x$ (by the strict concavity of $\pi(p,c)$). The latter inequality is due to the facts that $\lim_{t\to p-c}\left[\pi_p(p^n,c) - D(p)\right] = (p-c)D'(p) < 0$ while $\lim_{t\to p-c}\Theta(p,t) = 0$. These inequalities prove that $\underline{\delta}$ reaches a peak at $t_0 = t_0(p, \pi^*_{ROW}) \in T^x$, where $\underline{\delta}_t = 0$.

Next, note that

$$\underline{\delta}_{tt}\big|_{t=t_0} = \frac{\left[\frac{1}{2}\pi^*_{ROW} + \pi(p, c+t)\right]\pi_{pp}(p^n, c)}{\Omega^2} < 0$$

due to the strict concavity of $\pi(p,c)$ in p. Naturally, this inequality implies that $\underline{\delta}$ is strictly quasi-concave in t and attains a unique maximum at t_0. As emphasized in part $(b.i)$, $\underline{\delta} \to \frac{1}{2}$ as $t \to 0$ and as $t \to t_x$.[5] For clarity, we illustrate $\underline{\delta}$ as a function of t in Fig. 2(a) for three values of p: when $p = p^m$ (the pink curve) and when $p < p^m$ (the blue and green curves).

[2] By part (b) of Lemma A1, strict concavity requires $\pi(p,c) - \pi(p^n,c) < \pi_p(p^n,c)(p-p^n)$. $\Theta > 0$ follows by rearranging terms.

[3] The second property is due to the fact that $\pi_p(p^m,c) = D(p^m) + (p^m - c)D'(p^m) = 0$.

[4] Although we have defined $\underline{\delta}^x(p)$ as a function of p we also write it as a function of t along t_x (as shown in the figure) because $t_x = p - c$.

[5] One can confirm the validity of this claim by taking the limits of $\underline{\delta}$ in (6).

Part ($b.ii$) can be proven by differentiating $\underline{\delta}$ in (6) to obtain $\partial\underline{\delta}/\partial\pi^*_{ROW} = -\frac{1}{2}\Psi/\Omega^2 \leq 0$. This inequality follows from the fact that $\Psi \geq 0$ by Lemma 1(b).

Part ($b.iii$) can be established by noting that $d\underline{\delta}^0(p)/dp = \underline{\delta}_p + \underline{\delta}^0_t(dt_0/dp) = \underline{\delta}_p > 0$. This expression is positive because the second term vanishes (recall that $\underline{\delta}_t = 0$ at $t = t_0$) and $\underline{\delta}_p > 0$ by Lemma 3. We can visualize $\underline{\delta}^0$ in the (t, δ) space by relating prices and trade costs along $t_0(p)$. Since $\underline{\delta}^0_t = 0$ defines t and p implicitly, we may differentiate $\underline{\delta}^0_t$ and use the implicit function theorem to obtain $dp/dt|_{t=t_0} = -\underline{\delta}^0_{tt}/\underline{\delta}^0_{tp}$. Taking derivatives and utilizing the fact that $\underline{\delta}^0_t = 0$, we find

$$\underline{\delta}^0_{tt} = \frac{1}{\Omega^2}\left[\frac{1}{2}\pi^*_{ROW} + \pi(p, c+t)\right]\pi_{pp}(p^n, c) < 0.$$

To obtain $\underline{\delta}^0_{tp}$, define $\Upsilon \equiv \frac{1}{2}\pi^*_{ROW} + \pi(p, c) - (p-c)\pi_p(p^n, c) + t^2 D'(p^n)$ and note that we can use Υ to rewrite $\underline{\delta}^0_t = 0$ as

$$\underline{\delta}^0_t = \frac{1}{\Omega^2}\left[-D(p)\Upsilon + \frac{1}{2}\pi^*_{ROW}\pi_p(p^n, c)\right] = 0.$$

Clearly, $\underline{\delta}^0_t = 0$ requires the collusive price to be such that $\Upsilon > 0$. Additionally, the concavity of the per-period profit in price, along with our focus on $p \in (p^n, p^m]$, imply $\Upsilon_p = \pi_p(p, c) - \pi_p(p^n, c) < 0$. Thus, differentiation of $\underline{\delta}^0_t$ with respect to p delivers

$$\underline{\delta}^0_{tp} = \frac{1}{\Omega^2}\left[-D'(p)\overset{(+)}{\Upsilon} - D(p)\overset{(-)}{\Upsilon}_p\right] > 0.$$

On the basis of the above analysis we conclude that $dp/dt|_{t=t_0} > 0$, which implies that $\underline{\delta}^0$ can be viewed as being positively related to t, as indicated by the black schedule in Fig. 2(a).

Part ($b.iv$) follows from $\underline{\delta}_p > 0$ (Lemma 3), the strict quasi-concavity of $\underline{\delta}$ in t just established, and $\underline{\delta} < 1$ (which holds because $\Pi^a > \Pi^n$ for $p > p^n$). ||

S3: Proof of Proposition 6 (linear demand)

Suppose $\delta \in \left(\frac{1}{2}, \underline{\delta}_{\max}\right)$ and $t \in (t_1, t_2)$, as stated in the proposition, so that p is defined implicitly from $\underline{\delta}(p, t, \pi^*_{ROW}) = \delta$. Label this solution $p^c(t, \delta, \pi^*_{ROW})$. Now recall from the proof of Proposition 3($b.iii$) that $dp^c/d\pi^*_{ROW} = -\underline{\delta}_{\pi^*_{ROW}}/\underline{\delta}_p$ and, for simplicity (but no loss of generality), set $\beta = 1$ in the (linear) demand function. It is easy for one to show that

$$dp^c/d\pi^*_{ROW} = \frac{p^c - c - t}{2\left[\frac{1}{2}\pi^*_{ROW} + (p^c - c - t)^2\right]} > 0. \quad \text{(S.5)}$$

Thus, for any given $t \in (t_1, t_2)$, the lower the value of π^*_{ROW}, the higher the impact of a marginal increase in π^*_{ROW} on price p^c. In particular, $\lim_{\pi^*_{ROW} \to 0} (dp^c/d\pi^*_{ROW}) = \frac{1}{2(p^c-c-t)}$ for $p^c - c - t > 0$.

From (7), the impact of π^*_{ROW} on welfare can be rewritten as

$$dV^c/d\pi^*_{ROW} = \frac{1}{2} - (p^c - c)\,(dp^c/d\pi^*_{ROW}) = \frac{1}{2}\left[1 - \frac{(p^c - c)\,(p^c - c - t)}{\frac{1}{2}\pi^*_{ROW} + (p^c - c - t)^2}\right]. \quad \text{(S.6)}$$

Provided $p^c - c - t > 0$, we would have

$$\lim_{\pi^*_{ROW} \to 0} (dV^c/d\pi^*_{ROW}) = -\frac{t}{2\,(p^c - c - t)} < 0.$$

When π^*_{ROW} is sufficiently small to start with, an increase in π^*_{ROW} reduces welfare. This is so because the increase in p^c (a pro-collusive effect), brought about by the increase in π^*_{ROW}, outweighs the direct (and positive through the increase in income) effect of π^*_{ROW} on welfare, causing welfare to fall.

The key question is whether $p^c - c - t > 0$, which is equivalent to requiring $p^c > p^n$ (since $p^n = c + t$). But this is always true for the values of δ and t considered for $\pi^*_{ROW} > 0$ (even if π^*_{ROW} is infinitesimal). This proves part (*a*).

From (S.5) one can see that $dp^c/d\pi^*_{ROW} \to 0$ as $\pi^*_{ROW} \to \infty$ (provided $p^c - c - t > 0$). But the highest possible level of p^c is p^m. Under our assumptions on δ and t, there must exist a level of π^*_{ROW}, labeled $\widetilde{\pi}^*_{ROW}$, that ensures $p^c = p^m$. So $p^c < p^m$ for $\pi^*_{ROW} < \widetilde{\pi}^*_{ROW}$ and $p^c = p^m$ otherwise. This demonstrates that, if π^*_{ROW} is already sufficiently large, further increases in π^*_{ROW} will unambiguously raise welfare because the adverse pro-collusive effect will vanish as $p^c = p^m$. This proves part (*b*).

It remains to record the closed forms that underlie the discussion above and Fig. 3. Let p^c_0 and t^c_0 denote the price and trade cost that solve the system $\underline{\delta}(p, t, \pi^*_{ROW}) = \delta$ and $\underline{\delta}_t\,(p, t, \pi^*_{ROW}) = 0$ simultaneously—the peak of the constrained path. Solving (after some tedious but straightforward algebra, and rejecting the irrelevant roots) yields:

$$p^c_0 = \frac{c\delta + 2\alpha\left(\delta - \frac{1}{2}\right) + 2\sqrt{\delta\left(\delta - \frac{1}{2}\right)\pi^*_{ROW}}}{3\left(\delta - \frac{1}{3}\right)} \quad \text{(S.7a)}$$

$$t^c_0 = \frac{2\,(\alpha - c)\,\delta\left(\delta - \frac{1}{2}\right) + (1 - \delta)\sqrt{\delta\left(\delta - \frac{1}{2}\right)\pi^*_{ROW}}}{3\delta\left(\delta - \frac{1}{3}\right)} > 0, \quad \text{(S.7b)}$$

for $p^c_0 < p^m$ and $\delta \in \left(\frac{1}{2}, 1\right)$.[6] With the help of the above values in p^c_0 and t^c_0, one can also

[6] It can be shown that $p^c_0 < p^m$ if $\pi^*_{ROW} < \widetilde{\pi}^*_{ROW} = \frac{(\alpha-c)^2(1-\delta)^2}{16\delta\left(\delta-\frac{1}{2}\right)}$.

show that

$$p_0^c - c - t_0^c = \sqrt{\left(\frac{\delta - \frac{1}{2}}{\delta}\right)\pi^*_{ROW}} > 0$$

$$p_0^c - c = \frac{2}{3}\left[\frac{(\alpha - c)\left(\delta - \frac{1}{2}\right) + \sqrt{\delta\left(\delta - \frac{1}{2}\right)\pi^*_{ROW}}}{\delta - \frac{1}{3}}\right] > 0.$$

The positive signs of the expressions above are due to the facts that $\delta \in \left(\frac{1}{2}, 1\right)$ and $\pi^*_{ROW} > 0$. Moreover, $p_0^c - c - t_0^c$ becomes infinitesimally small as $\pi^*_{ROW} \to 0$. Since $\lim_{\pi^*_{ROW} \to 0} t_0^c > 0$, $p_0^c - c$ in (S.6) remains bounded away from zero as $\pi^*_{ROW} \to 0$. Now note that differentiation of p_0^c in (S.7a) with respect to π^*_{ROW} gives

$$dp_0^c/d\pi^*_{ROW} = \frac{1}{3\left(\delta - \frac{1}{3}\right)}\sqrt{\frac{\delta\left(\delta - \frac{1}{2}\right)}{\pi^*_{ROW}}} > 0.$$

The above expression reveals that the price response diverges when profit opportunities in *ROW* are very small: $dp_0^c/d\pi^*_{ROW} \to \infty$ as $\pi^*_{ROW} \to 0$. This is the pro-collusive force behind part (*a*): with $p_0^c - c$ bounded away from zero, the price channel in (S.6) dominates the income channel at low π^*_{ROW}.

For additional clarity, we illustrate the importance of trade costs t and profits in *ROW* π^*_{ROW} for welfare V^* in Fig. 3. Parts (*a*) and (*b*) become clear for fixed $t \in (t_1, t_2)$ as we travel along the surface with increases in π^*_{ROW} from 0 and on. The surface also depicts the response of welfare to increases in π^*_{ROW} along t_0^c (S.7b). ||

The general-demand statement of part (*a*) (Proposition 5)—and its sharpening to a global statement under weak convexity of demand—is proved in Appendix B below.

Appendix B: Demand Curvature and Proposition 5

This appendix proves part (*a*) of Proposition 5: B.1 gives the local result—near the collapse threshold, for any admissible demand—and B.2 its global completion under weak convexity of demand. Part (*b*) needs no curvature restriction and is established in the text accompanying Proposition 5.

B.1. Local result (general demand).

Using $\partial\underline{\delta}/\partial\pi^*_{ROW} = -\frac{1}{2}\Psi/\Omega^2$ (proof of Proposition 1(*b.ii*)) and $\underline{\delta}_p$ from (S.1), the implicit function theorem gives, for any demand satisfying (A2),

$$\frac{dp^c}{d\pi^*_{ROW}} = -\frac{\underline{\delta}_{\pi^*_{ROW}}}{\underline{\delta}_p} = \frac{D(p^n) - D(p^c)}{2D(p^c)\left[-\frac{1}{2}\pi^*_{ROW}\frac{D'(p^c)}{D(p^c)} + D(p^n)\Lambda(p^c,t)\right]} > 0.$$

Substituting the above expression into (7) and letting $\pi^*_{ROW} \to 0$ gives

$$\lim_{\pi^*_{ROW}\to 0} \frac{dV^c}{d\pi^*_{ROW}} < 0 \iff S(p^c,t) > 0,$$

where

$$S(p,t) \equiv [D(p^n) - D(p)]\left[D(p) - (p-c)D'(p)\right] + D(p^n)\,(p-p^n)\,D'(p).$$

Two properties of $S(\cdot)$ deliver a local version of part (a) for general demand. First, $S(p^n,t) = 0$ and $\partial S/\partial p|_{p=p^n} = t\,D'(p^n)^2 > 0$, so $S > 0$ for all p^c in a right-neighborhood of p^n. Second, since the criterion is evaluated as $\pi^*_{ROW} \to 0$, the relevant constrained price is that of the Proposition 2 benchmark, which falls to the Bertrand-Nash level as t rises to the collapse threshold $t_2^0 \equiv t_2|_{\pi^*_{ROW}=0}$ (the threshold of Proposition 2 at $\pi^*_{ROW} = 0$, where $\underline{\delta} = \underline{\delta}^x$; Fig. 1). This threshold is exactly the $\pi^*_{ROW} \to 0$ limit of the price-trough location t_0 of the U-shaped constrained path obtained with *ROW* profits (Fig. 2(b))—not of the larger trade cost at which the ICC slackens—so the mechanism operates near the trough of that path. Hence, for *any* demand satisfying (A2), increases in π^*_{ROW} from low initial levels reduce host welfare whenever t is close to t_2^0: part (a) of Proposition 5 holds for general demand—a *local* statement, on a neighborhood of t_2^0; the *global* version, covering the entire constrained interval, follows for weakly convex demand in B.2 below. Linearity is what delivers the global statement: with $D(p) = \alpha - \beta p$, direct computation gives $S = \beta t\,[D(p^n) - D(p)] > 0$ on all of $(p^n, p^m]$. In contrast, one can verify that S may turn negative for prices well above p^n when demand is sufficiently concave (e.g., $D(p) = 1 - p^2$ at low t). The global result (B.2) below shows that this is the only possibility: weak convexity of demand delivers the global characterization of part (a), and every strictly concave demand generates failures at low t. $\quad ||$

*B.2. Global result (weak convexity).*The local argument above admits a sharp global completion. For $p > p^n$ and $t \in [0, \bar{t}\,)$, define the *demand chord* over $[p^n, p]$:

$$C(p,t) \equiv \frac{D(p^n) - D(p)}{p - p^n} > 0.$$

By Lemma 1(b), $\Psi(p,t) = t\,C(p,t)\,(p - p^n)$, so the chord measures the ICC-relaxing power of profits in *ROW* per unit of price (recall $\partial\underline{\delta}/\partial\pi^*_{ROW} = -\frac{1}{2}\Psi/\Omega^2$), while the local slope

$D'(p)$ governs both the deadweight loss per unit of price in (7) and, through Λ in (S.2), the ICC-tightening cost of a higher price. The function $S(\cdot)$ defined above aggregates exactly these two objects:

Lemma B1 *(Chord identity)*: *For any strictly decreasing, differentiable demand function, any $p > p^n$, and any $t \geq 0$,*

$$S(p,t) = (p - p^n)\left[D(p)\left(C(p,t) + D'(p)\right) - t\,C(p,t)\,D'(p)\right]. \tag{S.8}$$

Proof: Substitute $D(p^n) = D(p) + C(p,t)\,(p - p^n)$ and $p - c = (p - p^n) + t$ into the definition of $S(\cdot)$ and collect terms. ||

Identity (S.8) has four immediate consequences. First, letting $p \downarrow p^n$ (so that $C \to -D'(p^n)$ and $D(p) \to D(p^n)$) recovers in one line the local result used above: $S/(p - p^n) \to t\,D'(p^n)^2 > 0$. Second, because $D(p) - tD'(p) > 0$, it delivers a necessary and sufficient version of the immiserization criterion:

$$S(p,t) > 0 \iff C(p,t) > \frac{-D(p)D'(p)}{D(p) - tD'(p)} \;\big(< -D'(p) \text{ for } t > 0\big):$$

the chord must exceed the local slope discounted by the factor $D/(D - tD')$. Third, and most importantly:

Theorem B1 *(Convexity delivers the global characterization)*: *Suppose demand satisfies (A2) and is weakly convex on $[p^n, p^m]$. Then $S(p,t) > 0$ for every $p \in (p^n, p^m]$ and every $t \in (0, \bar{t}\,)$. Hence, for any such demand, part (a) of Proposition 5 holds at* every *$t \in (t_1, t_2)$: increases in π^*_{ROW} from sufficiently low initial levels reduce host welfare.*

Proof: Weak convexity means $-D'$ is non-increasing, so by the mean value theorem $C(p,t) = -D'(\xi)$ for some $\xi \in (p^n, p)$, whence $C(p,t) \geq -D'(p)$ and the first term inside the brackets of (S.8) is non-negative; the second term is strictly positive for $t > 0$. (If demand is strictly convex, $C > -D'(p)$ and $S > 0$ at $t = 0$ as well.) ||

Fourth, failures require concavity—and concavity produces them generically at low trade costs. By the second consequence above, $S \leq 0$ forces $C(p,t) < -D'(p)$: the chord must fall short of the local slope, which is impossible under weak convexity. Conversely, for strictly concave demand, letting $t \to 0$ in (S.8) gives $S/(p - p^n) \to D(p)\,[C(p,0) + D'(p)] < 0$ at *every* $p > p^n$: every strictly concave demand violates the global characterization at sufficiently low trade costs. This explains why the counterexample $D = 1 - p^2$ requires low t, and why the failure margin shrinks as t grows. Linear demand, with $C = -D' = \beta$, is the knife-edge of the convex class: (S.8) collapses to $S = \beta t\,[D(p^n) - D(p)]$, the expression obtained above, which is positive only by virtue of $t > 0$.

The constant-elasticity case now follows as a corollary—of particular interest because a maintained assumption of profit concavity excludes it (see the Introduction), while (A2) does not.

Corollary B1 *(Proposition 6 under constant elasticity of demand)*: *Let $D(p) = Ap^{-\varepsilon}$ with $\varepsilon > 1$, so that $\gamma(p) = (\varepsilon+1)/\varepsilon \in (1,2)$, (A2) holds, and demand is strictly convex, with $p^m = \varepsilon c/(\varepsilon - 1)$ and $\bar{t} = c/(\varepsilon-1)$. Then $S(p,t) > 0$ on all of $(p^n, p^m] \times [0, \bar{t}\,)$, and* Proposition 6 *holds verbatim with "linear" replaced by "constant-elasticity"; in particular, the immiserization region at low π^*_{ROW} is the entire constrained interval (t_1, t_2).*[7]

Finally, two threshold objects that are implicit for general demand become explicit here. For any admissible demand, L'Hôpital's rule applied to $\underline{\delta}$ at $\pi^*_{ROW} = 0$ gives the junction limit $\lim_{p \downarrow p^n} \underline{\delta} = \left[2 - \frac{t}{p^n}\varepsilon(p^n)\right]^{-1}$, where $\varepsilon(p) \equiv -pD'(p)/D(p)$; this recovers the limits $\frac{1}{2}$ (as $t \to 0$) and 1 (as $t \to \bar{t}$) reported after Lemma 3. Under constant elasticity of demand, $\varepsilon(p) = \varepsilon$ is constant, so the collapse threshold t_2 solves $\varepsilon t/\,(c+t) = 2 - 1/\delta$ in closed form:

$$t_2 = \frac{c\,(2\delta - 1)}{1 + \delta\,(\varepsilon - 2)},$$

which is decreasing in ε: more elastic demand narrows the window of trade costs over which constrained collusion—and with it the immiserization mechanism of Proposition 5(a)—operates. Numerical computation of the full model under constant elasticity of demand confirms the results above and reveals, in addition, that $|dV^c/d\pi^*_{ROW}|$ at low π^*_{ROW} rises by an order of magnitude as $t \uparrow t_2$: improvements in *ROW* profitability are most damaging to cartel hosts precisely where collusion is closest to collapsing on its own.

Appendix C: The Punishment Convention

Section 2 adopts grim-trigger reversion to the Bertrand-Nash equilibrium of the static price game. This appendix records why, what changes under the maximal punishments of Abreu (1986, 1988), and in what sense the two conventions bracket the space of credible punishments.

C.1. What maximal punishments would do. Under maximal punishments—assuming such

[7] A direct proof, independent of Theorem B1, is also available: substituting $D'(p) = -\varepsilon D(p)/p$ into $S(\cdot)$ and writing $x \equiv p/p^n \geq 1$ and $s \equiv t/p^n$ gives $S = A^2\,(p^n)^{-2\varepsilon}\,x^{-\varepsilon-1}G(x)$ with $G(x) = \left(1 - x^{-\varepsilon}\right)[x + \varepsilon\,(x-1) + \varepsilon s] - \varepsilon\,(x-1)$, which is increasing in s; and $\varphi(x) \equiv G(x)|_{s=0}$ satisfies $\varphi(1) = \varphi'(1) = 0$ and $\varphi''(x) = \varepsilon\,(\varepsilon+1)\,x^{-\varepsilon-2}\,[\varepsilon - (\varepsilon-1)\,x]$, so φ' rises from zero on $\left(1, \frac{\varepsilon}{\varepsilon-1}\right)$ and then falls monotonically to its limit $1 > 0$; hence $\varphi > 0$ on $(1, \infty)$.

punishments are themselves credible—punishment payoffs would be driven to the minmax value of zero *regardless* of domestic market structure, and the limit-pricing profits at the heart of our benchmark would vanish. Because the deviation payoff is unchanged (see the discussion of optimal deviations in Section 2) and the punishment payoff is zero, the domestic-monopoly model under maximal punishments coincides with the $n = 1$ instance of the analysis of Section 5, whose formulas remain valid there with $\kappa = \frac{1}{2}$. A sufficiently patient cartel ($\delta \geq \frac{1}{2}$) would then sustain full monopoly in every market at every trade cost level, while an impatient one would find fragmentation pro-collusive. Far from revealing a fragility, this observation identifies the source of our reversal: the key determinant is whether punishment-phase profits depend on trade costs. Under Nash reversion they do so only when each host market is monopolized; under maximal punishments they never do.

C.2. Why Nash reversion. Two considerations support it. First, credibility: reversion to the static Bertrand-Nash equilibrium requires no intertemporal incentives to carry out, whereas holding a deviator to its minmax of zero under domestic monopoly requires the punisher to price below its delivered cost $c + t$ in the deviator's market—a stick that is itself incentive compatible only for sufficiently patient firms (Abreu, 1986, 1988) and that is vulnerable to renegotiation, since all parties would jointly gain from abandoning it. Second, evidence: documented cartel breakdowns and price wars resemble reversion to competitive pricing rather than sustained below-cost warfare. Note, finally, that the issue arises only under domestic monopoly: for $n \geq 2$, Nash reversion already attains the minmax, so the two conventions coincide and Section 5 is unaffected.

C.3. The bracketing argument. Let Π^{pun} denote the per-period punishment payoff under an arbitrary credible convention, so that the minimum discount factor sustaining an agreement is $\underline{\delta} = \left(\Pi^d - \Pi^a\right) / \left(\Pi^d - \Pi^{pun}\right)$. Since $\Pi^d > \Pi^a$ for every agreement above the punishment price, $\underline{\delta}$ is increasing in Π^{pun}: harsher punishments sustain more agreements. Under domestic monopoly the admissible range of credible punishment payoffs is $\Pi^{pun} \in [0, tD(c+t)]$, with the upper end attained by Nash reversion (Sections 3 and 4) and the lower end by maximal punishments, which—as shown in C.1—reproduce Section 5 read at $n = 1$. The set of sustainable agreements under *any* credible punishment convention is therefore nested between those of the two cases this paper fully characterizes. Whatever the harshest credible punishment turns out to be, the model with optimal penal codes is bracketed by results already in hand.