Managerial Compensation with Keeping up with the Joneses Agents

(Temporary Title)

Extremely Preliminary: please do not circulate without permission.

Peter M. DeMarzo
Stanford University and NBER

Ron Kaniel
University of Rochester, and CEPR

November 8, 2013
Preamble

The objective of our work is to understand the equilibrium contracting implications of having managers with Keeping up with the Joneses incentives. The lack of empirical evidence for relative performance pay is some what of a puzzle. The puzzle is exacerbated by the fact that empirically it has been shown that not only doesn’t compensation decline with peer firms performance, it in fact increases in peer performance.

We show that assuming managers have Keeping up with the Joneses incentives allows a simple rationalization for this evidence.

Our analysis focuses on three related models:

1. a setting with a single principle employing multiple managers.

2. a setting with multiple principle-agent pairs, in which each principle is responsible for writing the contract of her agent. Furthermore, in determining the optimal contract of a given firm it is assumed the effort choice of managers of other firms are taken as given, and consequently so is their income distribution.

3. similar to the second, but relaxes the assumption that other firms managers’ actions are taken as given when determining a firm’s contract. Specifically, in determining a manager’s contract the firm realizes that since managers have Keeping up with the Joneses preferences its compensation structure will impact effort choices of peer managers, which in turn will affect the effort choice of its manager.

In addition to showing our basic point, our setting allows us to derive testable implications on a few dimensions:

- We compare the pay performance and relative pay performance sensitivity of a division manager within a conglomerate to that of manager managing a stand alone firm that is similar to the conglomerate division.

- We analyze how do equilibrium contracts vary as one varies the degree of competition in the relevant industry. In our simple setting we use the number of firms as the proxy for competition.

- We derive testable predictions as to how contract structure changes as one varies output correlation across firms and the riskiness of the firms operations.

This draft contains the solution of the model, the different propositions, and a comprehensive set of comparative statics analysis. These results constitute the basis for the paper. The draft also contains a list of related papers, at this point without any associated discussion.

The paper itself will be written soon.....
1 $n + 1$ Agents

$q_i$ is the output of agent $i$, which depends on the agent’s effort choice.

$$q_i = a_i + \epsilon_i$$  \hspace{2cm} (1)

where $\epsilon_i \sim N(0, \sigma^2)$ have a pairwise correlation of $\rho \geq 0$.

Let

$$\bar{q}_{i-} = \frac{1}{n} \sum_{j \neq i} q_j$$

Contracts

$$w_i = m_i + x_i q_i + y_i \bar{q}_{i-}$$  \hspace{2cm} (2)

where,

$$\bar{w}_{i-} = \frac{1}{n} \sum_{j \neq i} w_j$$

$$\bar{\epsilon}_{i-} = \frac{1}{n} \sum_{j \neq i} \epsilon_j$$

So that

$$Cov[\epsilon_1, \epsilon_{i-}] = \rho \sigma^2$$

$$Var[\epsilon_{i-}] = \frac{1}{n^2} (n \sigma^2 + n(n - 1) \rho \sigma^2) = \frac{1 + (n - 1) \rho}{n}$$

and agents $i$’s utility functions become

$$-E[e^{-2\lambda(w_i - \delta \bar{w}_{i-} - \psi(a_i))}]$$

observe that

$$w_i = m_i + x_i (a_i + \epsilon_i) + y_i \frac{1}{n} \sum_{j \neq i} (a_j + \epsilon_j)$$
Consider a symmetric equilibria, and solve for the reaction function of the contract for a given principle-agent pair, taking all the other contracts as given.

W.l.o.g we solve for the reaction function of principle-agent pair 1, and to facilitate a simple comparison to the case with only two principle agent pair we denote the contracts of other agents by $x_2, y_2$ and the corresponding agent action by $a_2$.

### 1.1 Agents Optimization

Agents have keeping up with the Joneses exert effort $a_i$ to maximize

$$
\max_{a_i} -E[e^{-2\lambda(w_i-\delta\bar{w}_i-\psi(a_i))}],
$$

where $\psi(a) = \frac{1}{2r}a^2$.

subject to the IR constraint

$$-E[e^{-2\lambda(w_i-\delta\bar{w}_i-\psi(a_i))}] \geq \bar{u}
$$

Throughout we make the following natural assumption

**Assumption 1**

$$-1 < \delta < 1$$

Using the symmetry we can simplify to

$$w_1 = m_1 + x_1(a_1 + \epsilon_1) + y_1(a_2 + \bar{\epsilon}_1)$$

$$\bar{w}_{1-} = m_2 + x_2a_2 + x_2\bar{\epsilon}_1 + y_2\left(\frac{1}{n}(a_1 + \epsilon_1) + \frac{n-1}{n}(a_2 + \bar{\epsilon}_1)\right)$$
Which implies that

\[ w_1 - \delta \bar{w}_1 = m_1 - \delta m_2 + \left( x_1 - \frac{\delta y}{n} \right) a_1 + \left( y_1 - \delta \left( x_2 + \frac{(n-1)y}{n} \right) \right) a_2 \]

\[ + \left( x_1 - \frac{\delta y}{n} \right) \epsilon_1 + \left( y_1 - \delta \left( x_2 + \frac{(n-1)y}{n} \right) \right) \bar{\epsilon}_1 \]

So that

\[ E[w_1 - \delta \bar{w}_1] = m_1 - \delta m_2 + \left( x_1 - \frac{\delta y}{n} \right) a_1 + \left( y_1 - \delta \left( x_2 + \frac{(n-1)y}{n} \right) \right) a_2 \]

and

\[ Var[w_1 - \delta \bar{w}_1] = \left[ \left( x_1 - \frac{\delta y}{n} \right)^2 + 2 \left( x_1 - \frac{\delta y}{n} \right) \left( y_1 - \delta \left( x_2 + \frac{(n-1)y}{n} \right) \right) \rho \right. \]

\[ \left. + \left( y_1 - \delta \left( x_2 + \frac{(n-1)y}{n} \right) \right)^2 \frac{1 + (n-1)\rho}{n} \right] \sigma^2 \]

Using the MGF for normally distributed random variables, the agent maximization can be written as

\[ \max_{a_1} \left[ 2\lambda (E[w_1 - \delta \bar{w}_1] - \psi(a_1)) - \frac{1}{2}4\lambda^2 Var[w_1 - \delta \bar{w}_1] \right] \]

or written slightly differently as

\[ \max_{a_1} 2\lambda [E[w_1 - \delta \bar{w}_1] - \psi(a_1) - 2\lambda Var[w_1 - \delta \bar{w}_1]] \]

Taking the other agents action as given, and eliminating terms that are independent of the agents action \( a_1 \), the maximization reduces to

\[ \max_{a_1} \left[ (x_1 - \frac{\delta y}{n}) a_1 - \frac{1}{2k} a_1^2 \right] \]

with a solution

\[ a_1^* = k \left( x_1 - \frac{\delta y}{n} \right) \]

So that an agent’s action depends both on the sensitivity of his payoff to his output and the sensitivity of other agents payoffs to his output.

Also, by substituting in the \( a_1^*, a_2^* \) for \( a_1, a_2 \) we get

\[ E[w_1 - \delta \bar{w}_1] = m_1 - \delta m_2 + k \left( x_1 - \frac{\delta y}{n} \right)^2 + k \left( y_1 - \delta \left( x_2 + \frac{(n-1)y}{n} \right) \right) \left( x_2 - \frac{\delta (y_1 + \frac{(n-1)y}{n})}{n} \right) \]
1.2 Single Principle with \( n + 1 \) Agents

Optimal contract is given by

**Proposition 1** Let

\[
R_o = (n + \delta) \left( (1 + (n - 1)\rho) + (1 - \rho)(1 + n\rho)\frac{2\sigma^2\lambda}{k} \right)
\]

The optimal equilibrium contract is given by

\[
x_o = \frac{(1 - \delta \rho) + (n - 1)(1 - \delta)(1 + n\rho)}{R_o}
\]

\[
y_o = \frac{n((\delta - \rho) - (n - 1)\rho(1 - \delta))}{R_o}
\]

\[
a_o = \frac{k(1 - \delta)(n + \delta)(1 + (n - 1)\rho)}{R_o}
\]

**Comment:** for \( x_o \) and \( y_o \), \( k \), \( \sigma \) and \( \lambda \) enter together through the term \( \frac{2\sigma^2\lambda}{k} \), which impacts the scaling term \( R_o \).

**Lemma 1**

1. \( y_o \)
   - \( y_o < 0 \) iff \( \delta < \frac{n\rho}{1+(n-1)\rho} \).
   - increases in \( \delta \), and decreases in \( \rho \),
   - varying \( \sigma^2 \), \( \lambda \), and \( k \):
     - \( \rho < 1 \): if \( \delta > \frac{n\rho}{1+(n-1)\rho} \) decreases in \( \sigma^2 \) and \( \lambda \), and increases in \( k \). Otherwise, increases in \( \sigma^2 \) and \( \lambda \), and decreases in \( k \).
     - \( \rho = 1 \): independent of \( \sigma^2 \), \( \lambda \), and \( k \).

2. \( x_o \)
   - \( x_o > 0 \) Let \( \delta_1 = \frac{(n+n^2)\frac{2\sigma^2\lambda}{k}}{1+(n+n^2)\frac{2\sigma^2\lambda}{k}} \) and \( \delta_2 = \frac{(n-1)n}{(n-1)n+1} \)
   - decreases in \( \delta \),
   - varying \( \rho \)
     - \( \delta \leq 0 \): increases in \( \rho \).
     - \( \delta > 0 \): If \( \delta > \frac{(n+n^2)\frac{2\sigma^2\lambda}{k}}{1+(n+n^2)\frac{2\sigma^2\lambda}{k}} \) decreases in \( \rho \); otherwise u-shaped in \( \rho \)
   - varying \( \sigma^2 \), \( \lambda \), and \( k \):
     - \( \rho < 1 \): decreases in \( \sigma^2 \) and \( \lambda \), and increases in \( k \).
     - \( \rho = 1 \): independent of \( \sigma^2 \), \( \lambda \), and \( k \).
3. \( x_o > y_o \) iff \( \delta < \frac{1+(2n-1)\rho}{n-1+1+n\rho} \) (note: from the condition if either \( n = 1 \) or \( \rho = 0 \) then \( x_o > y_o \))

4. \( a_o \)
   - \( a_o > 0 \)
   - decreases in \( \delta \), and increases in \( \rho \)
   - varying \( \sigma^2 \), and \( \lambda \):
     - \( \rho < 1 \): decreases in \( \sigma^2 \) and \( \lambda \),
     - \( \rho = 1 \): independent of \( \sigma^2 \), \( \lambda \).
   - increases in \( k \)

5. \( \frac{\mu_o}{x_o} \) increases in \( \delta \), decreases in \( \rho \), and is independent of \( \sigma^2 \), \( \lambda \), \( k \).

6. \( m_o \)
   - increases (decreases) in \( \delta \) if \( \frac{2\sigma^2 \lambda}{k} < (>) \frac{1-n\frac{\sigma^2}{1+n\rho}}{1+np} \). Note that the condition implies that at \( \rho = 1 \) always decreases in \( \delta \). Also, in the region that the cutoff \( \frac{1-n\frac{\sigma^2}{1+n\rho}}{1+np} \) is positive the cutoff is decreasing in \( \rho \).
   - increases in \( \rho \)
   - varying \( \sigma^2 \), and \( \lambda \):
     - \( \rho < 1 \): increases (decreases) in \( \sigma^2 \) and \( \lambda \) if \( \frac{2\sigma^2 \lambda}{k} < (>) \frac{3-n\frac{\sigma^2}{1+n\rho}}{1+np} \),
     - \( \rho = 1 \): independent of \( \sigma^2 \) and \( \lambda \).
   - increases in \( k \) if \( \frac{\sigma^2 \lambda}{k} \geq 1 \) or \( \rho > \frac{1}{1+n} \), and decreases in \( k \) if \( \frac{\sigma^2 \lambda}{k} < 1 \) and \( \rho \) sufficiently small.

7. principle’s welfare
   - decreases in \( \delta \), increases in \( \rho \)
   - varying \( \sigma^2 \), and \( \lambda \):
     - \( \rho < 1 \): decreases in \( \sigma^2 \) and \( \lambda \),
     - \( \rho = 1 \): independent of \( \sigma^2 \) and \( \lambda \).
   - increases in \( k \)

Lemma 2 As a function of \( n \)

1. \( y_o \):
• 0 < ρ < 1:

\[
\begin{align*}
0 & \leq \delta < \rho, \quad \text{decreasing} \\
\rho & < \delta, \quad \text{decreasing for } n \text{ sufficiently large} \\
\frac{1}{2} (1 - \sqrt{3}) & < \delta < 0, \quad \text{if } \rho > \frac{1}{(1 - \delta + \delta^2)} \quad \text{increasing} \\
\frac{1}{2} (1 - \sqrt{3}) & < \delta < 0, \quad \text{if } \rho < \frac{1}{(1 - \delta + \delta^2)} \quad \text{decreasing for } n \text{ sufficiently large} \\
\frac{1}{2} (1 - \sqrt{3}) & < \delta < -\frac{1}{3}, \quad \text{if } \rho > \frac{(1 + 2\delta - 2\delta^2)}{\delta^2} \quad \text{increasing}
\end{align*}
\]

• ρ = 0 (ρ = 1): increasing (decreasing).

2. \( x_0 \):

• 0 < ρ < 1:

\[
\begin{align*}
\delta & < -\frac{1 - \rho}{2\sigma_x^2 + 1 - \rho}, \quad \text{decreasing for } n \text{ sufficiently large} \\
\delta & > -\frac{1 - \rho}{2\sigma_x^2 + 1 - \rho}, \quad \text{increasing for } n \text{ sufficiently large} \\
\delta & = 0, \quad \text{increasing}
\end{align*}
\]

• ρ = 0 (ρ = 1): decreasing (increasing).

3. \( \frac{w}{x_0} \):

• ρ > 0:

\[
\begin{align*}
\rho & > \frac{\delta^2}{2 + \delta - 2\delta^2}, \quad \text{decreasing} \\
\rho & < \frac{\delta^2}{2 + \delta - 2\delta^2}, \quad \text{tent-shape}
\end{align*}
\]

• ρ = 0: increasing.

4. \( a_0 \):

• 0 < ρ < 1: increasing

• ρ = 0 and ρ = 1: independent.

5. \( m_0 \):

• 0 < ρ < 1: increasing

• ρ = 0 and ρ = 1: independent.

6. principle’s welfare per agent hired

• 0 < ρ < 1: increasing;

• ρ = 0 and ρ = 1: independent.

Proof. to be added
1.2.1 Limit as $n \to \infty$

**Corollary 1** The optimal equilibrium contract is given by

\[
\begin{align*}
x_o &= \frac{1 - \delta}{1 + (1 - \rho) \frac{2 \sigma^2 \lambda}{k}} \\
y_o &= -x_o \\
a_o &= k x_o \\
m_o &= \frac{k(1 - \delta)}{2} \frac{1}{1 + (1 - \rho) \frac{2 \sigma^2 \lambda}{k}} \\
Principle’s Welfare &= \frac{k(1 - \delta)}{2} \frac{1}{1 + (1 - \rho) \frac{2 \sigma^2 \lambda}{k}}
\end{align*}
\]

1.2.2 $w_i = m_i + u_i(q_i - \bar{q}) + v_i \bar{q}$

\[
\begin{align*}
u_o &= x_o \\
v_o &= x_o + y_o = \frac{(1 - \rho)(n + \delta)}{R_o} = \frac{1 - \rho}{(1 + (n - 1)\rho) + (1 - \rho)(1 + n\rho) \frac{2 \sigma^2 \lambda}{k}}
\end{align*}
\]

1.2.3 The $\delta = 0$ benchmark

Setting $\delta = 0$ in Proposition 1 yields

**Corollary 2** When $\delta = 0$

\[
R_o = n \left( (1 + (n - 1)\rho) + (1 - \rho)(1 + n\rho) \frac{2 \sigma^2 \lambda}{k} \right),
\]

and the optimal equilibrium contract is given by

\[
\begin{align*}
x_o &= \frac{n^2 \rho + n}{R_o} \\
y_o &= \frac{-n^2 \rho}{R_o} \\
a_o &= \frac{kn(1 + (n - 1)\rho)}{R_o}
\end{align*}
\]

**comment:** The comparative statics for this case are embedded as part of the comparative statics in Lemmas 1 and 2.
1.3 \( n + 1 \) Principle-Agent Pairs when actions of other agents are taken as given

1.3.1 Reaction function of Principle 1

Proposition 2 Taking the contract of all other agents as given, the optimal contract that principle 1 gives to agent 1 is given by

\[
\begin{align*}
x_{1g}^{\text{react}} &= \frac{n(1 + (n - 1)\rho) + y_2\delta(1 - \rho)(1 + n\rho)\frac{2\sigma^2\lambda}{k}}{n((1 + (n - 1)\rho) + (1 - \rho)(1 + n\rho)\frac{2\sigma^2\lambda}{k})} \\
y_{1g}^{\text{react}} &= -\frac{np}{(1 + (n - 1)\rho) + (1 - \rho)(1 + n\rho)\frac{2\sigma^2\lambda}{k}} + x_2\delta \\
&\quad + y_2\delta \left(1 - \frac{1}{n}\left(1 + (n - 1)\rho + (1 - \rho)(1 + n\rho)\frac{2\sigma^2\lambda}{k}\right)\right)
\end{align*}
\]

immediate observations:

- \( x_{1g}^{\text{react}} \) is independent of \( x_2 \)
- \( x_{1g}^{\text{react}} \) increases in \( y_2 \) iff \( \delta > 0 \)
- \( y_{1g}^{\text{react}} \) increases in \( x_2 \) and \( y_2 \) iff \( \delta > 0 \)

comment: when \( \rho = 0 \) reaction function reduces to

\[
\begin{align*}
x_{1g}^{\text{react}} &= \frac{n + y_2\delta\frac{2\sigma^2\lambda}{k}}{n(1 + \frac{2\sigma^2\lambda}{k})} \\
y_{1g}^{\text{react}} &= x_2\delta + y_2\delta \left(1 - \frac{1}{n}\right)
\end{align*}
\]

1.3.2 Equilibrium Contracts

Proposition 3 Let

\[
R_g = (1 - \delta)R_o + \delta(\delta - \rho - (n - 1)\rho(1 - \delta))
\]

The optimal equilibrium contract is given by

\[
\begin{align*}
x_g &= \frac{(1 - \delta\rho) + (n - 1)(1 - \delta)(1 + n\rho)}{R_g} \\
y_g &= \frac{n((\delta - \rho) - (n - 1)\rho(1 - \delta))}{R_g} \\
a_g &= \frac{k(1 - \delta)(n + \delta)(1 + (n - 1)\rho)}{R_g}
\end{align*}
\]
• Comparing the solution to the one with one principle it is easy to see that \( x_g, y_g, \) and \( a_g \) differ from their counterparts \( x_o, y_o, \) and \( a_o \) in that the scaling term \( R_g \) replaces the scaling term \( R_o. \)

• A direct calculation shows that \( R_g > 0, \) and that \( R_g < R_o \) iff \( \delta > 0. \)

**Lemma 3** When \( \delta = 0 \) the equilibrium coincides with the case where we have one principle with multiple agents.

**Proof.** By inspection. ■

**Lemma 4**

1. \( y_g \)
   - \( y_g < 0 \) iff \( \delta < \frac{np}{1+(n-1)\rho}; \) comment: same sign as \( y_o. \)
   - increases in \( \delta, \) and decreases in \( \rho; \) at \( \rho = 1 \) independent of \( \delta. \)
   - varying \( \sigma^2, \lambda, \) and \( k: \)
     - for \( \rho < 1: \) if \( \delta > \frac{np}{1+(n-1)\rho} \) decreases in \( \sigma^2 \) and \( \lambda, \) and increases in \( k. \) Otherwise, increases in \( \sigma^2 \) and \( \lambda, \) and decreases in \( k. \)
     - for \( \rho = 1: \) independent of \( \sigma^2, \lambda, \) and \( k. \)
     - comment: same comparative statics as in one principle case.

2. \( x_g \)
   - \( x_g > 0 \)
   - varying \( \delta \)
     - \( 0 \leq \rho < 1: \) U-shaped in \( \delta, \) where the \( \delta \) at which the minimum is attained is at \( \delta = 0 \) when \( \rho = 0 \) and increases in \( \rho. \)
     - \( \rho = 1: \) independent of \( \delta. \)
   - tent shaped in \( \rho, \) where the peek is at \( \rho^* \) which solves \( \delta = \frac{1}{1+\left\{\frac{(1-\rho)^2}{(n\rho(2+(n-1)\rho)}\right\}^{1/2}}. \) At \( \delta = 0 \) \( \rho^* \) is 0 (i.e., \( x_g \) decreases in \( \rho). \) Also, \( \rho^* \) increases in \( \delta \) and decreases in \( n. \)
   - varying \( \sigma^2, \lambda, \) and \( k: \)
     - \( \rho < 1: \) decreases in \( \sigma^2 \) and \( \lambda, \) and increases in \( k. \)
     - \( \rho = 1: \) independent of \( \sigma^2, \lambda, \) and \( k. \)
     - comment: same comparative statics as in one principle case.

3. \( x_g > y_g \) iff \( \delta < \frac{1}{1+\left\{\frac{(1-\rho)^2}{(2n\rho(2+(n-1)\rho)}\right\}^{1/2}}. \) For \( n = 1 \) \( x_g > y_g. \) The cutoff is U-shaped in \( n, \) and for \( n > 1 \) the cutoff is increasing in \( \rho. \)
4. $a_g$

- $a_g > 0$; at the limit as $\delta \to 1$ $a_g \to 0$.
- varying $\delta$:
  \[
  \begin{cases}
  \rho = 1, & \text{increasing in } \delta \\
  0 \leq \rho < 1, & \text{tent-shape in } \delta \text{(the max occurs at a } \delta^* > 0, \text{ where } \delta^* \text{ increases in } \rho) 
  \end{cases}
  \]
- increases in $\rho$
- varying $\sigma^2$, and $\lambda$
  - $\rho < 1$: decreasing in $\sigma^2$, and $\lambda$;
  - $\rho = 1$: independent of $\sigma^2$, $\lambda$.
  - comment: same as in one principle case.
- increasing in $k$, comment: same as in one principle case.

5. $\frac{y_g}{x_g} = \frac{y_o}{x_o}$

6. $m_g$ (seems too messy to get full analytical characterization)

- varying $\delta$
  - $\rho < 1$ and $n \geq 2$
    \[
    \begin{cases}
    \frac{2\sigma^2 \lambda}{k} > \frac{1-(1+n)\rho}{(1-\rho)(1+n\rho)} & \text{tent-shaped, for } \delta \leq 0 \text{ subregion increasing.} \\
    \text{otherwise,} & \text{u-shaped, for } \delta \leq 0 \text{ subregion decreasing.}
    \end{cases}
    \]
  - $\rho < 1$ and $n = 1$:
    * similar characterization as the $n \geq 2$ for the $\delta \leq 0$ range. $f$ in addition $\frac{2\sigma^2 \lambda}{k} > \frac{\rho}{(1-\rho^2)}$ or $\frac{1}{4} \frac{\rho}{(1-\rho^2)} > \frac{2\sigma^2 \lambda}{k}$ (in the mid range derivative may switch sign more than once-seems messy to check analytically)
  - $\rho = 1$: increasing
- varying $\rho$: when $n = 1$ a sufficient condition for it to increase in $\rho$ is $\delta > -0.215$(seems messy to sign for region $\delta < -0.215$ or for general $n$)
- varying $\sigma^2$, and $\lambda$
  - $\rho < 1$: if $\rho \geq \frac{3n(1-\delta) + \delta(3-5\delta)}{n(3-n) + (3-n-n^2)\delta - 2(1+n)\delta^2}$ decreasing, otherwise tent-shaped.
  - $\rho = 1$: independent.
- varying $k$: if $\rho > \frac{1}{1+n}$ increases, otherwise tent-shaped.

7. principle’s welfare (seems too messy to get full analytical characterization)
• varying $\delta$,
  - $\rho = 1$: If $n \geq 7$ decreases, otherwise tent-shape where max is at a $\delta^*(n) < 0$.
• varying $\rho$
  - $\rho = 0$: decreasing in $\rho$
  - $\rho = 1$: increasing in $\rho$ iff $\frac{\sigma^2}{k}(1-\delta)(n+\delta)(n-2(1+n)\delta) - \delta^2 > 0$ (note that a sufficient condition for it to be decreasing is $\delta > \frac{n}{2(1+n)}$).
• varying $\sigma^2$ and $\lambda$
  - for $\rho = 1$: independent of $\sigma^2$ and $\lambda$.
  - $\frac{1}{n+1} < \rho < 1$: there exist thresholds $\frac{1}{2} \leq \delta_1 \leq \delta_2 < 1$ such that
    \[
    \begin{cases}
    \delta < \delta_1, & \text{decreasing} \\
    \delta_1 < \delta < \frac{1}{2}, & \text{tent-shaped} \\
    \frac{1}{2} < \delta < \delta_2, & \text{increasing} \\
    \delta_2 < \delta, & \text{u-shaped}
    \end{cases}
    \]
  - $\rho < \frac{1}{n+1}$: there exist thresholds $\frac{1}{2} \leq \delta_1 \leq \delta_2 < 1$ such that
    \[
    \begin{cases}
    \delta < \frac{1}{2}, & \text{decreasing} \\
    \frac{1}{2} < \delta < \delta_1, & \text{u-shaped} \\
    \delta_1 < \delta < \delta_2, & \text{increasing} \\
    \delta_2 < \delta, & \text{u-shaped}
    \end{cases}
    \]

For $n \leq 5$, there exists a $\rho^*(n) < \frac{1}{n+1}$ such that for $\rho \leq \rho^*(n)$ $\frac{1}{2} = \delta_1 = \delta_2$; for $n > 5$ $\frac{1}{2} < \delta_1 < \delta_2$
• varying $k$:
  Let $\delta^* = \frac{(n-n\rho+n^2\rho)}{(-1+(n+1)\rho+2(n-n\rho+n^2\rho))}$
  - $\rho < \frac{1}{n+1}$:
    \[
    \begin{cases}
    \delta < \frac{1}{2}, & \text{increases} \\
    \frac{1}{2} < \delta < \delta^*, & \text{u-shape} \\
    \delta^* < \delta, & \text{decreases}
    \end{cases}
    \]
  - $\rho > \frac{1}{n+1}$:
    \[
    \begin{cases}
    \delta^* > \delta, & \text{increases} \\
    \delta^* < \delta < \frac{1}{2}, & \text{tent-shape} \\
    \delta > \frac{1}{2}, & \text{decreases}
    \end{cases}
    \]
\( \rho = 1 \): decreases in \( k \) iff \( \delta > \frac{n}{2n+1} \)

**Lemma 5** As a function of \( n \)

1. \( y_g \):
   - \( 0 < \rho < 1 \):
     \[
     \begin{cases}
     0 < \delta < \rho, & \text{decreasing} \\
     \rho < \delta, & \text{either decreasing or tent shape, } \rho \text{ sufficiently small tent-shape} \\
     \delta < 0, & \rho \text{ or } \delta \text{ sufficiently small, tent shape}
     \end{cases}
     \]
   - \( \rho = 0 \) \( (\rho = 1) \): increasing (decreasing)

2. \( x_g \):
   - \( 0 < \rho < 1 \): Let \( \delta_1 = \frac{3\rho^2 - \rho \sqrt{8+24\rho+17\rho^2}}{2(1+3\rho+\rho^2)} < 0 > -1 \), \( \delta_2 = \frac{3\rho^2 + \rho \sqrt{8+24\rho+17\rho^2}}{2(1+3\rho+\rho^2)} \leq 1 \).
     \[
     \begin{cases}
     \delta_1 < \delta < \delta_2, & \text{increasing} \\
     \text{otherwise, } & \text{u-shaped}
     \end{cases}
     \]

   comment: \( \delta_1 \) is decreasing in \( \rho \) and \( \delta_2 \) is increasing in \( \rho \).
   - \( \rho = 0 \) \( (\rho = 1) \): negative (positive).

3. \( y_g \): the same as in the one principle case.

4. \( a_g \):
   - \( \delta < 0 \): decreasing
   - \( 0 < \rho < 1 \):
     if \( \delta < \frac{2\sigma^2 \lambda (1-\rho)}{k} \) for large enough \( n \) increasing.
     \[
     \begin{cases}
     \delta > \rho, & \text{increasing} \\
     \frac{k}{\sigma^2 \lambda} \text{ small,} & \text{increasing} \\
     \delta < \frac{2\rho^2}{1+2\rho-\rho^2} \text{ and } \frac{k}{\sigma^2 \lambda} \text{ large, } & \text{u-shaped}
     \end{cases}
     \]

   If \( \delta > \frac{2\sigma^2 \lambda (1-\rho)}{k} \) for large enough \( n \) decreasing.
     \[
     \begin{cases}
     \delta > \frac{2\rho^2}{1+2\rho-\rho^2}, & \text{u-shaped} \\
     \frac{k}{\sigma^2 \lambda} \text{ small,} & \text{u-shaped} \\
     \text{otherwise,} & \text{decreasing}
     \end{cases}
     \]
• $\rho = 0$ and $\rho = 1$: increasing (decreasing)

Proof. to be added ■

1.3.3 Limit as $n \to \infty$

Corollary 3 A principle’s reaction function is given by

$$x_{1g}^{\text{react}} = \frac{1}{1 + (1 - \rho) \frac{2\sigma^2 \lambda}{k}}$$

$$y_{1g}^{\text{react}} = -\frac{1}{1 + (1 - \rho) \frac{2\sigma^2 \lambda}{k}} + (x_{2g} + y_{2g}) \delta$$

The optimal equilibrium contract is given by

$$x_g = \frac{1}{1 + (1 - \rho) \frac{2\sigma^2 \lambda}{k}}$$

$$y_g = -x_g$$

$$a_g = kx_g$$

$$m_g = \frac{k}{2(1 - \delta)} \frac{1}{1 + (1 - \rho) \frac{2\sigma^2 \lambda}{k}}$$

Principle’s Welfare $$= \frac{k(1 - 2\delta)}{2(1 - \delta)} \frac{1}{1 + (1 - \rho) \frac{2\sigma^2 \lambda}{k}}$$

1.3.4 $w_i = m_i + u_i(q_i - \bar{q}) + v_i \bar{q}$

$$u_g = x_g$$

$$v_g = x_g + y_g = \frac{(n + \delta)(1 - \rho)}{R_g}$$

1.3.5 comparing across contract types

Lemma 6

1. $y_g > y_o$ iff $\frac{\delta(1 - \rho)}{\rho(1 - \delta)} > n$ (comment: for $n = 1$ condition reduces to $\delta > \rho$)

2. $x_g > x_o$ iff $\delta > 0$

3. $a_g > a_o$ iff $\delta > 0$

4. $m_g > m_o$ iff $\delta \left( \frac{2\sigma^2 \lambda}{k} - \frac{1 - (n + 1) \rho}{(1 - \rho)(1 + n\rho)} \right) > 0$
5. Principle’s welfare per-agent hired is higher when one principle hires all agents, as opposed to \( n + 1 \) principle-agent pairs.

**Proof.** To be inserted. ■
1.4 \( n + 1 \) Principle-Agent Pairs

1.4.1 Reaction function of Principle 1

**Proposition 4** Taking the contract of the other \( n \) agents as given the optimal contract that principle 1 gives to agent 1 is given

\[
x_{1m}^{react} = \frac{n(1 + (n-1)\rho) - x_2 \delta^2 \rho + y_2 \left( \delta(1-\rho)(1+n\rho) \frac{2\sigma^2}{k} - (1 - \frac{1}{n})\delta^2 \rho \right)}{n \left( 1 + (n-1)\rho + (1-\rho)(1+n\rho) \frac{2\sigma^2}{k} \right)}
\]

\[
y_{1m}^{react} = \frac{-n^2 \rho + x_2 \delta \left( \frac{k}{2\sigma^2} + n + \delta + (n-1)n\rho + n(1-\rho)(1+n\rho) \frac{2\sigma^2}{k} \right)}{n \left( 1 + (n-1)\rho + (1-\rho)(1+n\rho) \frac{2\sigma^2}{k} \right)}
\]

\[
y_{1m}^{react} + \frac{y_2 \delta \left( (1 - \frac{1}{n}) \frac{k}{2\sigma^2} + (1 - \frac{1}{n})(n + \delta) + (1 + (n-1)n)\rho + (n-1)(1-\rho)(1+n\rho) \frac{2\sigma^2}{k} \right)}{n \left( 1 + (n-1)\rho + (1-\rho)(1+n\rho) \frac{2\sigma^2}{k} \right)}
\]

Immediate observations

- \( y_{1m}^{react} \) increasing in both \( x_2 \) and \( y_2 \).
- \( x_{1m}^{react} \) decreasing in \( x_2 \).
- \( x_{1m}^{react} \) is increasing in \( y_2 \) iff \( \frac{k}{2\sigma^2} < \frac{(1-\rho)(1+n\rho)}{(1-\frac{1}{n})\delta \rho} \). For \( \delta < 0 \) decreasing in \( y_2 \). When \( \delta > 0 \) and \( \rho > 0 \) the RHS of this inequality decreases in \( \delta \) and \( \rho \), and is u-shaped as a function of \( n \).
- Comparing to the reaction function for the case with actions of others taken as given:

\[
x_{1m}^{react} = x_{1g}^{react} - \rho \frac{\delta^2 \left( x_2 + y_2(1 - \frac{1}{n}) \right)}{n \left( 1 + (n-1)\rho + (1-\rho)(1+n\rho) \frac{2\sigma^2}{k} \right)}
\]

\[
y_{1m}^{react} = y_{1g}^{react} + \left( 1 + \frac{k}{2\lambda \sigma^2} \right) \frac{\delta^2 \left( x_2 + y_2 (1 - \frac{1}{n}) \right)}{n \left( 1 + (n-1)\rho + (1-\rho)(1+n\rho) \frac{2\sigma^2}{k} \right)}
\]
Comment: in the case $\rho = 0$ the reaction function reduces to:

\[
\begin{align*}
x_{1m}^{\text{react}} &= \frac{n + y_2\delta 2\sigma^2 \lambda}{n \left(1 + \frac{2\sigma^2 \lambda}{k}\right)} \\
y_{1m}^{\text{react}} &= \frac{x_2 \delta \left(\frac{k \delta}{2\lambda \sigma^2} + n + \delta + n \frac{2\sigma^2 \lambda}{k}\right)}{n \left(1 + \frac{2\sigma^2 \lambda}{k}\right)} + \frac{y_2 \delta \left(\frac{k(n-1)\delta}{2\lambda \sigma^2} + (n-1)(n + \delta) + n + (n-1)n \frac{2\sigma^2 \lambda}{k}\right)}{n^2 \left(1 + \frac{2\sigma^2 \lambda}{k}\right)} \\
&= y_{1g}^{\text{react}} + \left(1 + \frac{k}{2\lambda \sigma^2}\right) \frac{\delta^2 (x_2 + y_2(1 - \frac{1}{n}))}{n \left(1 + \frac{2\sigma^2 \lambda}{k}\right)} = y_{1g}^{\text{react}} \frac{1 + \delta^2 \left(1 + \frac{k}{2\lambda \sigma^2}\right)}{n \left(1 + \frac{2\sigma^2 \lambda}{k}\right)}
\end{align*}
\]

1.4.2 Equilibrium Contracts

To find the equilibrium contracts we impose the condition $x_1 = x_2$ and $y_1 = y_2$ to get

Optimal contract is given by

Proposition 5 Let

\[
R_m = (1 - \delta)R_o - \delta^2 \left(1 - \frac{1}{n}\right) \frac{k}{2\lambda \sigma^2} + \frac{\delta(1 - \delta)(\delta - n^2 \rho)}{n}
\]

The optimal equilibrium contract is given by

\[
\begin{align*}
x_m &= \frac{(1 - \delta \rho) + (n - 1)(1 - \delta)(1 + n \rho) - \delta^2 \left(1 - \frac{1}{n}\right) \frac{k}{2\sigma^2 \lambda}}{R_m} \\
y_m &= \frac{n((\delta - \rho) - (n - 1)\rho(1 - \delta)) + \delta^2 \frac{k}{2\sigma^2 \lambda}}{R_m} \\
a_m &= \frac{k \left((1 - \delta)(\delta + n \rho) - \delta^2 \left(1 - \frac{1}{n}\right) \frac{k}{2\sigma^2 \lambda}\right)}{R_m}
\end{align*}
\]

Lemma 7 When $\delta = 0$ the equilibrium coincides with the case where we have one principal with multiple agents.

Proof. By inspection. ■

The following can be shown for $R_m$:

- When $n = 1 R_m > 0$
- For $\frac{k}{2\lambda \sigma^2} \leq 1$, $\delta \leq \frac{2(1 - \rho)}{3 - 2\rho}$ is a sufficient condition for $R_m > 0$. Specifically, the sufficient condition holds for $\delta \leq 0$.
- holding other parameters fixed, there exist a threshold $H > 0$ so that $R_m > 0$ if $\frac{k}{2\lambda \sigma^2} < H$
The comparative statics for the case \( n = 1 \) appears in detail in Lemma 11; in the subsection that provides the results for two principle agent pairs. The following lemma provides characterizations for \( n \geq 2 \).

**Comments Regarding Lemma 8:**

1. the lemma below provides partial analytical characterizations, some more partial than others, as in general it is too messy to get full analytical characterizations for this case.

2. the reported results ignore jumps at the discontinuity points.

3. unless specified otherwise, thresholds are parameter dependent (i.e., when varying a specific parameter, the thresholds potentially depend on all the other parameters).

4. keeping in mind that both \( \delta \) and \( \rho \) have a finite support, statements regarding \( \frac{k}{\sigma^2\lambda} \) being sufficiently small or large, for a given \( \delta \) or \( \rho \), can typically be extended to hold for the whole support of \( \delta \) or \( \rho \).

**Lemma 8**  
Assume \( n \geq 2 \)

1. \( y_m \)
   - varying \( \delta \)
     \[
     \begin{cases}
     \delta \geq 0, & \text{increasing} \\
     \delta < 0, & \begin{cases}
     1 > \rho, & \text{for each } \delta \text{ there exist a threshold } T > 0 \text{ such that increasing iff } \frac{k}{\sigma^2\lambda} < T \\
     \rho = 1, & \text{decreasing}
     \end{cases}
     \end{cases}
     \]
   - varying \( \rho \)
     \[
     \begin{cases}
     \delta > 0, & \begin{cases}
     \rho \text{ sufficiently small, decreasing} \\
     \rho \text{ intermediate levels of } \frac{k}{\sigma^2\lambda}, & \text{u-shape}
     \end{cases} \\
     \delta < 0, & \begin{cases}
     \rho \text{ sufficiently large, increasing}
     \end{cases}
     \end{cases}
     \]

\(^1\)The support of \( \delta \) is the open interval \((-1, 1)\). In some cases the relevant functions blow up at the boundary of the support. Thus, in some cases, one may not extended the local statement regarding \( \frac{k}{\sigma^2\lambda} \) to the whole support, but instead the precise statement should be that it is up to \( \epsilon \) from the boundary, where \( \epsilon \) can be chosen as small as we want.
• varying $\sigma^2$, $\lambda$, and $k$

\[
\left\{
\begin{array}{ll}
\rho < 1, & \delta > \frac{n\rho}{1-\rho+n\rho}, \text{ increasing in } \frac{k}{\sigma^2\lambda} \\
\rho = 1, \text{ increasing } & \text{otherwise, u-shaped in } \frac{k}{\sigma^2\lambda}
\end{array}
\right.
\]

2. $x_m$

• varying $\delta$
  - given $\delta$ for $\frac{k}{\sigma^2\lambda}$ sufficiently large locally decreasing
  - there exist a threshold $M(n,\rho)$ such that for $\delta > (<)M$ for $\frac{k}{\sigma^2\lambda}$ sufficiently small locally increasing (decreasing)
  - $\rho > 0$: increasing in the neighborhood of $\delta = 0$

• varying $\rho$
  - $\delta > 0$
    \[
    \left\{
    \begin{array}{ll}
    \frac{k}{\sigma^2\lambda} \text{ sufficiently large, increasing } & \frac{k}{\sigma^2\lambda} \text{ sufficiently small, u-shaped }
    \end{array}
    \right.
    \]

**comment:** For $\delta > \frac{n(n-1)}{1+n(n-1)}$ there is a cutoff between the two regions above. When $\delta < \frac{n(n-1)}{1+n(n-1)}$, have not been able to rule out a middle region where increasing-decreasing-increasing.

  - $\delta < 0$: there exists a cutoff $0 > M(n) > -1$ such that
    \[
    \left\{
    \begin{array}{ll}
    \delta < M(n), \text{ increasing for small values of } \rho & \frac{k}{\sigma^2\lambda} \text{ sufficiently small, increasing for small values of } \rho \\
    \delta > M(n), \text{ an intermediate range of } \frac{k}{\sigma^2\lambda}, \text{ u-shaped } & \frac{k}{\sigma^2\lambda} \text{ sufficiently large, increasing }
    \end{array}
    \right.
    \]

**comment:** in the two cases where have shown that increasing for small values of $\rho$ could either be increasing throughout or increasing-decreasing-increasing; have not been able to rule out the later thus far or find a numerical example for which it holds.

  - for $\rho$ sufficiently large increasing.

• varying $\sigma^2$, $\lambda$, and $k$

\[
\left\{
\begin{array}{ll}
\delta < 1 - n + n\rho, & \text{tent-shaped in } \frac{k}{\sigma^2\lambda} \\
\frac{n\rho}{1-\rho+n\rho} > \delta > 1 - n + n\rho, & \text{increasing-decreasing-increasing in } \frac{k}{\sigma^2\lambda} \\
\delta > \frac{n\rho}{1-\rho+n\rho}, & \text{tent-shaped in } \frac{k}{\sigma^2\lambda}
\end{array}
\right.
\]

20
3. \( a_m \)

- varying \( \delta \)
  - \( \rho > 0 \):
    * increasing in the neighborhood of \( \delta = 0 \)
    * \( \frac{k}{\sigma^2 \lambda} \) sufficiently large: increasing.
  - \( \rho \):
    * \( \delta > 0 \): exist a threshold \( M \) such that decreasing iff \( \frac{k}{\sigma^2 \lambda} < M \)
    * \( \delta < 0 \): exist threshold \( M_1 > M_2 \) such that decreasing iff \( M_1 > \frac{k}{\sigma^2 \lambda} > M_2 \)

- varying \( \rho \)

\[
\begin{align*}
\delta < 0, & \text{ exist a threshold } M \text{ such that if } \frac{k}{\sigma^2 \lambda} > M \text{ increasing, and otherwise u-shaped} \\
\delta > 0, & \begin{cases}
\frac{k}{\sigma^2 \lambda} \text{ sufficiently small}, & \text{increasing} \\
\text{exists a midrange of } \frac{k}{\sigma^2 \lambda}, & \text{tent-shaped} \\
\frac{k}{\sigma^2 \lambda} \text{ sufficiently large}, & \text{decreasing}
\end{cases}
\end{align*}
\]

comment: For \( \delta > 0 \) have not been able to rule out a range of \( \frac{k}{\sigma^2 \lambda} \) for which increasing-decreasing-increasing. Although, from running some numerical calculations it seems such a range does not exist.

For any \( \rho \) there exists a threshold such that when \( \frac{k}{\sigma^2 \lambda} \) is below the threshold locally increasing and when above it locally decreasing.

- varying \( \sigma^2 \), and \( \lambda \)

\[
\begin{align*}
\delta < 0, & \text{ decreasing} \\
\delta > 0, & \text{tent-shaped}
\end{align*}
\]

- varying \( k \) for \( \frac{k}{\sigma^2 \lambda} \) sufficiently small or sufficiently large increasing. comment: numerically, can produce cases where increasing throughout, as well as cases where increasing-decreasing-increasing.

4. \( \frac{y_m}{x_m} \)

- varying \( \delta \): increasing iff \( 2n(1-\rho)(1+n\rho) + \frac{k}{\sigma^2 \lambda} \delta(2-\delta) > 0 \)

comments: 1) inequality always holds for \( \delta > 0 \) 2) either increasing throughout or tent-shape.

- \( \rho = 1 \): increasing iff \( \delta > 0 \)
- \( \rho = 0 \): increasing iff \( 2n + \frac{k}{\sigma^2 \lambda} \delta > 0 \)
• varying $\rho$: increasing iff \[ \frac{\delta^3}{2n(1-\delta)(n+\delta)} > \frac{k}{\sigma^2\lambda} \]

**comment:** there exists a cutoff $0 < M < 1$, that depends on $n$ and $\frac{k}{\sigma^2\lambda}$, such that increasing iff $\delta > M$.

• varying $\sigma^2\lambda$, and $k$: increasing in $\frac{k}{\sigma^2\lambda}$

5. $m_m$:

• varying $\delta$: for any given $\delta$ for $\frac{k}{\sigma^2\lambda}$ sufficiently small or sufficiently large increasing.
• varying $\rho$:
  - $\rho < 1$: for any given $\delta$, for $\frac{k}{\sigma^2\lambda}$ sufficiently small increasing
  - $\rho = 1$: for any given $\delta$

\[
\delta > 0 \text{ or } n \geq 3, \quad \frac{k}{\sigma^2\lambda} \text{ sufficiently small increasing}
\]

\[
\begin{align*}
\delta < 1 - \frac{n}{\sqrt{2n-1}}, & \quad \text{for } \frac{k}{\sigma^2\lambda} \text{ sufficiently large decreasing} \\
n \geq 8, & \quad \text{for } \frac{k}{\sigma^2\lambda} \text{ sufficiently large increasing}
\end{align*}
\]

• varying $\sigma^2$, and $\lambda$: for any given $\delta$, for $\frac{k}{\sigma^2\lambda}$ sufficiently small decreasing.
• varying $k$: for any given $\delta$, for $\frac{k}{\sigma^2\lambda}$ sufficiently small or sufficiently large increasing.

6. principle’s welfare:

• varying $\delta$
  - $\rho < 1$:

\[
\begin{align*}
\delta > 1 - \frac{n}{\sqrt{2n-1}}, & \quad \text{for } \frac{k}{\sigma^2\lambda} \text{ sufficiently large decreasing} \\
\delta < 1 - \frac{n}{\sqrt{2n-1}}, & \quad \text{for } \frac{k}{\sigma^2\lambda} \text{ sufficiently large increasing} \\
n \geq 8, & \quad \text{for } \frac{k}{\sigma^2\lambda} \text{ sufficiently large decreasing}
\end{align*}
\]

**comment:** $0 > 1 - \frac{n}{\sqrt{2n-1}}$, and for $n \geq 8$ $-1 > 1 - \frac{n}{\sqrt{2n-1}}$.

  - $\rho = 1$:

\[
\begin{align*}
\frac{k}{\sigma^2\lambda} & \quad \text{sufficiently large, same as for } \rho < 1 \text{ comp-stat} \\
\delta > 0 \text{ and } n \geq 3, & \quad \text{for } \frac{k}{\sigma^2\lambda} \text{ sufficiently small increasing}
\end{align*}
\]

• varying $\rho$
\(-1 > \rho > 0:\)

\[
\begin{cases}
\delta > \frac{1}{2}, & \text{for } \frac{k}{\sigma^2 \lambda} \text{ sufficiently small decreasing} \\
\delta < \frac{1}{2}, & \text{for } \frac{k}{\sigma^2 \lambda} \text{ sufficiently small increasing} \\
\delta > -\frac{1+2n-\sqrt{1-2n+2n^3}}{2n}, & \text{for } \frac{k}{\sigma^2 \lambda} \text{ sufficiently large increasing} \\
\delta < -\frac{1+2n-\sqrt{1-2n+2n^3}}{2n}, & \text{for } \frac{k}{\sigma^2 \lambda} \text{ sufficiently large decreasing}
\end{cases}
\]

**Comment:** \(0 > -\frac{1+2n-\sqrt{1-2n+2n^3}}{2n}\) is monotone in \(n\) and for \(n > 7 - 1 > -\frac{1+2n-\sqrt{1-2n+2n^3}}{2n}\)

\(-\rho = 0:\)

\[
\begin{cases}
\frac{k}{\sigma^2 \lambda} \text{ sufficiently small, decreasing} \\
\frac{k}{\sigma^2 \lambda} \text{ sufficiently large, same as for } 1 > \rho > 0 \text{ comp-stat}
\end{cases}
\]

\(-\rho = 1:\)

\[
\begin{cases}
\frac{k}{\sigma^2 \lambda} \text{ sufficiently small, exists an } \frac{1}{2} > M > 0 \text{ where decreasing iff } \delta > M \\
\frac{k}{\sigma^2 \lambda} \text{ sufficiently large, same as for } 1 > \rho > 0 \text{ comp-stat}
\end{cases}
\]

- **Varying \(\sigma^2\), and \(\lambda\)**
  
  \(-\rho < 1:\)

\[
\begin{cases}
\frac{k}{\sigma^2 \lambda} \text{ sufficiently small, decreasing iff } \delta < \frac{1}{2}
\end{cases}
\]

\(-\rho = 1:\)

\[
\begin{cases}
\frac{k}{\sigma^2 \lambda} \text{ sufficiently large, decreasing} \\
\frac{k}{\sigma^2 \lambda} \text{ sufficiently small, increasing}
\end{cases}
\]

- **Varying \(k\)**
  
  \(-\rho < 1:\)

\[
\begin{cases}
\frac{k}{\sigma^2 \lambda} \text{ sufficiently large, increasing iff } \delta < \frac{n-1}{2n-1} \\
\frac{k}{\sigma^2 \lambda} \text{ sufficiently small, increasing iff } \delta < \frac{1}{2}
\end{cases}
\]

\(-\rho = 1:\)

\[
\begin{cases}
\frac{k}{\sigma^2 \lambda} \text{ sufficiently large, increasing iff } \delta < \frac{n-1}{2n-1} \\
\frac{k}{\sigma^2 \lambda} \text{ sufficiently small, increasing iff } (n^2 \delta + 2(-1 + \delta)\delta^2 + n^3(-1 + 2\delta)) < 0
\end{cases}
\]

**Comment:** A sufficient condition for \((n^2 \delta + 2(-1 + \delta)\delta^2 + n^3(-1 + 2\delta)) < 0\) is \(\delta < 0\), and for \((n^2 \delta + 2(-1 + \delta)\delta^2 + n^3(-1 + 2\delta)) > 0\) is \(\delta > \frac{1}{2}\).

For comparative statics with respect to \(n\) we can get a partial analytical characterization.
Lemma 9 As a function of $n$, holding other parameters fixed

1. $y_m$:
   - for $n$ sufficiently large
     \[
     \begin{cases}
     \rho > 0, & \text{decreasing} \\
     \rho = 0, & \text{increasing iff } \frac{k^2}{4n^4\lambda^2} < (1 - \delta)
     \end{cases}
     \]
   - for $\frac{k}{\sigma^2\lambda}$ sufficiently large: increasing
   - for $\frac{k}{\sigma^2\lambda}$ sufficiently small: there exists a threshold $\rho^* > 0$ such that increasing iff $\rho < \rho^*$

2. $x_m$:
   - for $n$ sufficiently large
     \[
     \begin{cases}
     \rho > 0, & \text{increasing} \\
     \rho = 0, & \text{decreasing}
     \end{cases}
     \]
   - for $\frac{k}{\sigma^2\lambda}$ sufficiently large: increasing iff $\delta > \rho$
   - for $\frac{k}{\sigma^2\lambda}$ sufficiently small: increasing iff $\frac{n\rho(1-\delta)}{1-\rho} > \delta > \frac{-n\rho}{1+\rho+n\rho}$ (comment: at $\rho = 0$ decreasing)

3. $\frac{y_m}{x_m}$:
   - for $n$ sufficiently large
     \[
     \begin{cases}
     \rho > 0, & \text{decreasing} \\
     \rho = 0, & \text{increasing iff } 1 > \frac{k}{2\sigma^2\lambda}
     \end{cases}
     \]
   - for $\frac{k}{\sigma^2\lambda}$ sufficiently large: increasing
   - for $\frac{k}{\sigma^2\lambda}$ sufficiently small: assuming $n \geq 2$ there exists a threshold $\rho^*$ such that increasing in $n$ iff $\rho < \rho^*$

4. $a_m$
   - for $n$ sufficiently large
     \[
     \begin{cases}
     \rho > 0, & \text{increasing iff } \frac{k}{2\sigma^2\lambda} \delta < (1 - \rho) \\
     \rho = 0, & \text{decreasing}
     \end{cases}
     \]
   - for $\frac{k}{\sigma^2\lambda}$ sufficiently large: increasing iff $\delta < 0$
• for $\frac{k}{\sigma^2 \lambda}$ sufficiently small:

$$\begin{cases} 1 > \rho > 0, & \text{increasing} \\ \rho = 0, & \text{increasing} \\ \rho = 1 \text{ and } n \geq 2, & \text{decreasing} \end{cases}$$

5. $m_m$

• for $n$ sufficiently large

$$\begin{cases} \rho > 0, & \text{increasing} \\ \rho = 0, & \text{decreasing} \end{cases}$$

• for $\frac{k}{\sigma^2 \lambda}$ sufficiently large, and assuming $n \geq 2$:

• for $\frac{k}{\sigma^2 \lambda}$ sufficiently small, and assuming $n \geq 2$:

$$\begin{cases} 1 > \rho > 0, & \text{increasing iff } 1 - 2\delta > 0 \\ \rho = 0, & \text{decreasing} \\ \rho = 1, & \text{increasing} \end{cases}$$

6. principle’s welfare

• for $n$ sufficiently large

$$\begin{cases} \rho > 0, & \text{increasing} \\ \rho = 0, & \text{decreasing} \end{cases}$$

• for $\frac{k}{\sigma^2 \lambda}$ sufficiently large, and assuming $n \geq 2$:

$$\begin{cases} \rho > 0, & \text{increasing} \\ \rho = 0, & \text{decreasing} \end{cases}$$

• for $\frac{k}{\sigma^2 \lambda}$ sufficiently small and assuming $n \geq 2$:

$$\begin{cases} \rho > 0, & \text{increasing} \\ \rho = 0, & \text{increasing} \end{cases}$$

Proof. to be added
1.4.3 Limit as $n \to \infty$

identical to the $n + 1$ principle agent pairs when actions are observable.

1.4.4 $w_i = m_i + u_i(q_i - \bar{q}) + v_i\bar{q}$

\[
\begin{align*}
  u_m &= x_m \\
  v_m &= x_m + y_m = \frac{n(n + \delta)(1 - \rho) + \frac{k\delta^2}{2\lambda\sigma}}{R_m}
\end{align*}
\]

1.4.5 comparing across contract types

The following can be shown:

- comparing $R_m$ to $R_o$
  \[\delta > 0 \Rightarrow R_m < R_o\]
  \[\text{for } n = 1: \delta < 0 \Rightarrow R_m > R_o\]
  \[\text{for } n > 1 \text{ and } \delta < 0: \frac{k}{2\lambda\sigma^2} < \frac{1}{2} \Rightarrow R_m > R_o, \text{ when } \frac{k}{2\lambda\sigma^2} \text{ sufficiently large } R_m < R_o\]

- comparing $R_m$ to $R_g$
  \[R_m = R_g - \delta^2 \left((1 - \frac{1}{n})(1 + \frac{k}{2\lambda\sigma^2}) + \left(\frac{\delta}{n} - \rho\right)\right)\]
  \[R_m < R_g \text{ iff } \delta > n\rho - (n - 1)(1 + \frac{k}{2\lambda\sigma^2})\]
  \[\text{for } n = 1 \text{ if } R_m < R_g \text{ iff } \delta > \rho\]

The optimal equilibrium contract can also be written as

\[
\begin{align*}
x_m &= \frac{R_o}{R_m} x_o - \frac{\delta^2 (1 - \frac{1}{n}) \frac{k}{2\sigma^2\lambda}}{R_m} = \frac{R_g}{R_m} x_g - \frac{\delta^2 (1 - \frac{1}{n}) \frac{k}{2\sigma^2\lambda}}{R_m} \\
y_m &= \frac{R_o}{R_m} y_o + \frac{\delta^2 \frac{k}{2\sigma^2\lambda}}{R_m} = \frac{R_g}{R_m} y_g + \frac{\delta^2 \frac{k}{2\sigma^2\lambda}}{R_m} \\
a_m &= \frac{R_o}{R_m} a_o - \frac{k \left(\delta^2 (1 - \frac{1}{n}) \frac{k}{2\sigma^2\lambda}\right)}{R_m} = \frac{R_g}{R_m} a_g - \frac{k \left(\delta^2 (1 - \frac{1}{n}) \frac{k}{2\sigma^2\lambda}\right)}{R_m}
\end{align*}
\]

Lemma 10

1. $y_m > y_g \iff R_m < 0$

2. $x_m > x_g \iff \delta(1 - \rho) < n\rho(1 - \delta) \text{ and } R_m > 0$, or $\delta(1 - \rho) > n\rho(1 - \delta) \text{ and } R_m < 0$
3. $a_m < a_g$ iff $\frac{-\delta}{(1-\delta)(n+\delta)} > \frac{2\sigma^2\lambda}{k}$ and $R_m > 0$, or $\frac{-\delta}{(1-\delta)(n+\delta)} < \frac{2\sigma^2\lambda}{k}$ and $R_m < 0$ (comment: note that for $\delta > 0$ $a_m < a_g$ iff $R_m < 0$)

4. $\frac{y_m}{x_m}$ vs $\frac{y_g}{x_g}$
   - $\frac{2\sigma^2\lambda}{k} < \frac{n-1}{n} - \frac{1}{2n^2\rho+2n-1}(1-\rho)$: there exists a cutoffs $-1 < \delta_1 < 0 < \delta_2$ such that $\frac{y_m}{x_m} > \frac{y_g}{x_g}$ iff $\delta < \delta_2$
   - $\frac{n-1}{n} - \frac{1}{2n^2\rho+2n-1}(1-\rho) < \frac{2\sigma^2\lambda}{k} < \frac{n-1}{n} - \frac{1}{1-\rho}$: there exists a cutoff $0 < \delta_2$ such that $\frac{y_m}{x_m} > \frac{y_g}{x_g}$ iff $\delta < \delta_2$
   - $\frac{n-1}{n} - \frac{1}{1-\rho} < \frac{2\sigma^2\lambda}{k}$: $\frac{y_m}{x_m} > \frac{y_g}{x_g}$

5. $m_m$ vs $m_g$: when $\frac{k}{\sigma^2\lambda}$ sufficiently small

\[
\begin{align*}
\begin{cases}
\quad n = 3, & m_m > m_g \\
\quad n = 2 \text{ and } \delta > \frac{2(1-\sqrt{11})}{5}, & m_m > m_g \\
\quad n = 2 \text{ and } \delta < \frac{2(1-\sqrt{11})}{5}, & \text{a cutoff } \rho^* \text{ such that } m_m > m_g \text{ iff } \rho > \rho^* \\
\quad n = 1, & m_m > m_g \text{ iff } \delta > 0
\end{cases}
\end{align*}
\]

when $\frac{k}{\sigma^2\lambda}$ sufficiently large $m_m < m_g$.

(comment: messy to try to get analytical characterization for the whole parameter space)

6. Principle’s welfare per-agent hired: for $\frac{k}{\sigma^2\lambda}$ sufficiently large Welfare$_m <$ Welfare$_g$, and when it is sufficiently small Welfare$_m <$ Welfare$_g$ iff $\delta < 0$;

Proof. To be inserted.
2 Two Agents

The solution for the case with two agents is obtained by setting \( n = 1 \)

2.1 Single Principle with Two Agents

Corollary 4 Let

\[
R_o = (1 + \delta) \left( 1 + (1 - \rho^2) \frac{2\sigma^2\lambda}{k} \right)
\]

The optimal equilibrium contract is given by

\[
\begin{align*}
x_o &= \frac{1 - \delta \rho}{R_o} \\
y_o &= \frac{\delta - \rho}{R_o} \\
a_o &= \frac{k(1 - \delta^2)}{R_o}
\end{align*}
\]

Comment: note that the condition \( \delta < \frac{\rho}{1(n-1)\rho} \) in Lemma 1 becomes \( \delta < \rho \), for \( n = 1 \).

2.1.1 \( w_i = m_i + u_i(q_i - q_j) + v_iq_j \)

\[
\begin{align*}
u_o &= x_o = \frac{1 - \delta \rho}{R_o} \\
v_o &= x_o + y_o = \frac{(1 + \delta)(1 - \rho)}{R_o} = u_o + \frac{\delta - \rho}{R_o}
\end{align*}
\]
2.2 2 Principle-Agent Pairs when actions of other agents are taken as given

2.2.1 Reaction function of Principle 1

Corollary 5 Taking the contract of all other agents as given, the optimal contract that principle 1 gives to agent 1 is given by

\[ x_{1g}^{react} = \frac{1 + y_2 \delta (1 - \rho^2) \frac{2\sigma^2 \lambda}{k}}{1 + (1 - \rho^2) \frac{2\sigma^2 \lambda}{k}} \]

\[ y_{1g}^{react} = -\frac{\rho}{1 + (1 - \rho^2) \frac{2\sigma^2 \lambda}{k}} + x_2 \delta + y_2 \delta \left( \frac{\rho}{1 + (1 - \rho^2) \frac{2\sigma^2 \lambda}{k}} \right) \]

2.2.2 Equilibrium Contracts

Corollary 6 Let

\[ R_g = (1 - \delta) R_o + \delta (\delta - \rho) \]

The optimal equilibrium contract is given by

\[ x_g = \frac{1 - \delta \rho}{R_g} \]

\[ y_g = \frac{\delta - \rho}{R_g} \]

\[ a_g = \frac{k (1 - \delta^2)}{R_g} \]

Proposition 6 The equilibrium is unique.

Proof. The reaction function in Corollary 5 can be written as \( v_1 = D + Av_2 \), where \( v_i = (x_i, y_i)^T \). Any equilibrium must satisfy \( v_2 = D + A(D + Av_2) \) or equivalently \( D + AD = (I - A^2) v_2 \). A direct calculation of the relevant determinants shows that \( I - A^2 \) is non-singular. ■
2.3 Two Principle-Agent Pairs

2.3.1 Reaction function of Principle 1

**Corollary 7** Taking the contract of agent 2 as given, the optimal contract that principle 1 gives to agent 1 is given by

\[
x_{1m}^{react} = \frac{1 - x_2 \delta^2 \rho + y_2 \delta (1 - \rho^2) \frac{2 \sigma^2 \lambda}{k}}{1 + (1 - \rho^2) \frac{2 \sigma^2 \lambda}{k}} - \rho \frac{x_2 \delta^2}{1 + (1 - \rho^2) \frac{2 \sigma^2 \lambda}{k}}
\]

\[
y_{1m}^{react} = \frac{-\rho + x_2 \delta \left( \delta \frac{k}{2 \sigma^2 \lambda} + (1 + \delta) \right) + y_2 \delta \rho}{1 + (1 - \rho^2) \frac{2 \sigma^2 \lambda}{k}} - \rho \frac{x_2 \delta^2}{1 + (1 - \rho^2) \frac{2 \sigma^2 \lambda}{k}} 
\]

immediate observations

- \(y_{1m}^{react}\) for \(\rho > 0\) increasing in \(x_2\) and \(y_2\); for \(\rho = 0\) constant in \(x_2\) and increasing in \(y_2\).

- \(x_{1m}^{react}\) for \(\rho > 0\) decreasing in \(x_2\) and increasing in \(y_2\); for \(\rho = 0\) decreasing in \(x_2\) and constant in \(y_2\).

- comparing to the reaction function for the case with actions of others taken as given:

\[
x_{1m}^{react} = x_{1g}^{react} - \delta^2 \rho \frac{x_2}{1 + (1 - \rho^2) \frac{2 \sigma^2 \lambda}{k}}
\]

\[
y_{1m}^{react} = y_{1g}^{react} + \delta^2 \frac{x_2 (1 + \frac{k}{2 \lambda \sigma^2})}{1 + (1 - \rho^2) \frac{2 \sigma^2 \lambda}{k}}
\]

in contrast to the case with \(n > 1\), the deviations from \(x_{1g}^{react}\) and \(y_{1g}^{react}\) do not depend on \(y_2\).

Observe that when \(\delta = 0\) the reaction contract does not depend on the other agent’s contract; when \(\rho = 0\) the reaction \(x_{1m}^{react}\) depends on \(y_2\), but not on \(x_2\); when \(\rho = 1\) the reaction \(x_{1m}^{react}\) depends on \(x_2\), but not on \(y_2\); if both \(\delta = 0\) and \(\rho = 0\) then \(y_{1m}^{react} = 0\).

2.3.2 Equilibrium Contracts

**Corollary 8** Let

\[
R_m = (1 - \delta) R_o + \delta (1 - \delta) (\delta - \rho)
\]
The optimal equilibrium contract is given by

\[
\begin{align*}
x_m &= \frac{1 - \delta \rho}{R_m} \\
y_m &= \frac{\left(\delta - \rho + \frac{k \delta^2}{2\sigma^2 \lambda}\right)}{R_m} \\
a_m &= \frac{k \left(1 - \delta^2 - \frac{k \delta^2}{2\sigma^2 \lambda}\right)}{R_m}
\end{align*}
\]

**Proposition 7** The equilibrium is unique.

**Proof.** Similar to the proof of Proposition 6, with a bit more work required to show that the relevant matrix is non-singular. ■

Note that \(\delta < 1 \Rightarrow R_m > 0\)

**Lemma 11** Let \(1 > \delta^*_1\) be the solution to \(\frac{\delta^3}{1 - \delta^2} = \frac{2 \sigma^2 \lambda}{k}\), \(\delta^*_2\) be the solution to \(\frac{\delta^3}{1 + \delta} = \frac{2 \sigma^2 \lambda}{k} + \left(\frac{2 \sigma^2 \lambda}{k}\right)^2\), (comment: both \(\delta^*_1\) and \(\delta^*_2\) are increasing in \(\frac{2 \sigma^2 \lambda}{k}\))

1. \(y_m\)

- \(y_m < 0\) iff \(\frac{\sigma^2 \lambda}{k} \left(-1 - \sqrt{1 + \frac{2 \rho k}{\sigma^2 \lambda}}\right) < \delta < \frac{\sigma^2 \lambda}{k} \left(-1 + \sqrt{1 + \frac{2 \rho k}{\sigma^2 \lambda}}\right) < 1\)

(comment: \(\frac{\sigma^2 \lambda}{k} \left(-1 - \sqrt{1 + \frac{2 \rho k}{\sigma^2 \lambda}}\right) > -1\) iff \(\frac{1}{2(1 + \rho)} > \frac{\sigma^2 \lambda}{k}\)

- varying \(\delta\):
  - \(1 > \rho\):
    - there exists a \(\tilde{\delta} < 0\) such that for \(\delta > \tilde{\delta}\) increases;
    - when \(\frac{\sigma^2 \lambda}{k} < 1\) decreasing for \(\delta < -\frac{127}{226}\);
    - There exists a cutoff, that depends on \(\rho\), so that for \(\frac{k}{\sigma^2 \lambda}\) below the cutoff increasing throughout.
  - \(\rho = 0\): if \(\frac{\sigma^2 \lambda}{k} > 1\) increasing throughout.
  - \(\rho = 1\): u-shaped (minimum at \(\delta = 0\))

- varying \(\rho\)
  \[
  \begin{cases}
  \delta \leq \delta^*_1, & \text{decreasing} \\
  \delta^*_1 < \delta < \min[1, \delta^*_2], & \text{u-shape} \\
  \delta^*_2 \leq \delta, & \text{increasing}
  \end{cases}
  \]

- varying \(\sigma\), \(\lambda\), and \(k\):
  - \(\delta > \rho\): increasing in \(\sigma\) and \(\lambda\), and decreasing in \(k\).
– $1 > \rho > \delta$: u-shaped in $\sigma$ and $\lambda$, and tent-shaped in $k$
– $\rho = 1$: decreasing in $\sigma$ and $\lambda$, and increasing in $k$.

2. $x_m$

- $x_m > 0$
- varying $\delta$:
  - $1 > \rho$
    $$\frac{k}{2\sigma^2 \lambda} < \frac{2(1-\rho^2)}{3+\rho}, \quad \text{u-shaped}$$
    $$\text{otherwise, increasing-decreasing-increasing}$$
  - $\rho = 1$ : tent-shaped

(comment: at $\delta = 0$ decreasing when $\rho > 0$ and flat when $\rho = 0$)

- varying $\rho$
  $$\begin{cases} 
  0 > \delta > -\frac{2\sigma^2 \lambda}{k}, & \text{increasing} \\
  \delta > 0 \text{ or } -\frac{2\sigma^2 \lambda}{k} > \delta, & \frac{\delta^2}{(2(1-\delta))} > \frac{2\sigma^2 \lambda}{k}, \quad \text{decreasing} \\
  \text{otherwise, } \frac{\delta^2}{(2(1-\delta))} > \frac{2\sigma^2 \lambda}{k}, \quad \text{u-shape}
  \end{cases}$$

- decreasing in $\sigma$ and $\lambda$, and increasing in $k$

3. $x_m - y_m$

Let $\hat{\delta}_1 = (1 + \rho)\frac{\sigma^2 \lambda}{k} \left(-1 - \sqrt{1 + \frac{2}{\frac{k}{1+\rho} \sigma^2 \lambda}}\right)$, $\hat{\delta}_2 = (1 + \rho)\frac{\sigma^2 \lambda}{k} \left(-1 + \sqrt{1 + \frac{2}{\frac{k}{1+\rho} \sigma^2 \lambda}}\right) < 1$ then

$x_m - y_m > 0$ iff $\hat{\delta}_1 < \delta < \hat{\delta}_2$. (comment: $\hat{\delta}_1 < -1$ iff $\frac{1}{(1+\rho)} > \frac{\sigma^2 \lambda}{k}$)

4. $\frac{y_m}{x_m}$

- varying $\delta$: if $\frac{k}{2\sigma^2 \lambda} > \frac{1-\rho^2}{2+\rho}$ increasing, otherwise u-shaped (comment: when u-shaped the minimum occurs a negative $\delta$, apart for where $\rho = 0$ in which case it occurs at $\delta = 0$)

- varying $\rho$:
  $$\begin{cases} 
  \delta < \delta_1^*, & \text{decreasing} \\
  \delta = \delta_1^*, & \text{independent} \\
  \text{otherwise, } \delta_1^* < \delta < \delta_2^*, & \text{increasing}
  \end{cases}$$

- For $\delta \neq 0$ decreasing in $\sigma$, and $\lambda$ and increasing in $k$; at $\delta = 0$ independent of $\sigma$, $\lambda$ and $k$.

5. $a_m$

- $a_m > 0$ iff $\delta < \delta_1^*$
• varying $\delta$: tent-shaped
• varying $\rho$:
  \[
  \begin{cases}
  \delta < 0, & \frac{\delta}{2(1+\delta)} \frac{k}{2\sigma^2 \lambda} < -1, \quad \text{decreasing} \\
  \text{otherwise,} & \quad \text{tent-shaped} \\
  \delta > 0, & \delta < \delta^*_1, \quad \text{increasing} \\
  \delta^*_1 < \delta, \quad \text{decreasing}
  \end{cases}
  \]
• varying $\sigma$ and $\lambda$: if $\delta \leq 0$ decreasing, otherwise, tent-shaped.
• varying $k$: if $\delta \leq 0$ increasing, otherwise tent-shaped.

(note that for $\delta > 0$ tent-shaped both in $k$ and in $\sigma$ and $\lambda$)

6. $m_m$(seems too messy to get a comprehensive analytical characterization)

• varying $\delta$: for a given $\delta$, for $\frac{k}{\sigma^2 \lambda}$ sufficiently large or sufficiently small locally increasing in $\delta$.
• varying $\rho$: for a given $\rho$, for $\frac{k}{\sigma^2 \lambda}$ sufficiently large or sufficiently small locally increasing in $\rho$.
• varying $\sigma$, $\lambda$: for $\sigma$ ($\lambda$) sufficiently small if $\delta > 0$ ($\delta < 0$) decreasing (increasing), and when sufficiently large decreasing.
• varying $k$: for $k$ sufficiently close to zero increasing and sufficiently large if $\delta > 0$ ($\delta < 0$) increasing (decreasing)

7. principle’s welfare (seems too messy to get a comprehensive analytical characterization)

• varying $\delta$: for a given $\delta$, for $\frac{k}{\sigma^2 \lambda}$ sufficiently small decreasing in $\delta$ and for $k$ sufficiently large if $\delta > 0$ ($\delta < 0$) locally decreasing (increasing) in $\delta$.
• varying $\rho$: for a given $\rho$, for $\frac{k}{\sigma^2 \lambda}$ sufficiently small increasing in $\rho$ iff $1 - 2\delta < 0$, for $\frac{k}{\sigma^2 \lambda}$ sufficiently large locally increasing in $\rho$.
• varying $\sigma$ and $\lambda$: for $\sigma$ ($\lambda$) sufficiently small increasing, and when $\sigma$ ($\lambda$) sufficiently large decreasing iff $1 - 2\delta > 0$.
• varying $k$: for $k$ sufficiently close to zero increasing iff $1 - 2\delta > 0$ and for $k$ sufficiently large decreasing.

Proof. To be added. ■

Note that when $\delta = 0$, $y_m < 0$ and $a_m > 0$. Also, when $\rho = 0$, $y_m > 0$.  

33
2.3.3 \[ w_i = m_i + u_i(q_i - q_j) + v_iq_j \]

\[
u_m = x_m = \frac{1 - \delta \rho}{R_m}
\]

\[
v_m = x_m + y_m = \frac{(1 + \delta)(1 - \rho) + \frac{k\delta^2}{2\sigma^2\lambda}}{R_m}
\]

2.3.4 comparing across contract types

comparing the equilibrium contracts in the case with one principle and two agents to the case with two principle agent pairs we get

Lemma 12

1. \( y_m \) vs \( y_o \)
   \[ \bullet \delta < 0: y_m > y_o \]
   \[ \bullet \delta > 0: \]
   \[ - \frac{k}{\sigma^2\lambda} \text{ sufficiently large, holding other parameters fixed, } y_m > y_o \]
   \[ - \frac{k}{\sigma^2\lambda} \text{ sufficiently small holding other parameters fixed, } y_m > y_o \text{ iff } \delta > \rho \]
   \[ - \text{ for } \delta \text{ sufficiently large, holding other parameters fixed, } y_m > y_o \]
   \[ - \rho = 0: y_m > y_o \]
   \[ - \rho = 1: \text{ there exist a threshold } \bar{\delta} > 0 \text{ such that } y_m > y_o \text{ iff } \delta > \bar{\delta} \]

2. \( x_m > x_o \) iff \( \delta > 0 \)

3. \( a_m \) vs \( a_o \)
   \[ \bullet \delta > 0: \text{there exist a threshold } \tilde{\delta} > 0 \text{ such that } a_m > a_o \text{ iff } \delta < \tilde{\delta} \]
   \[ \bullet \text{ Taking other parameters as fixed, there exists a threshold } T \text{ such that } a_m > a_o \text{ iff } \frac{k}{\sigma^2\lambda} < T \]
   \[ \bullet \delta > 0: \text{ a sufficient condition for } a_m < a_o \text{ is } \frac{k}{\sigma^2\lambda} < 1 \]

4. \[ \frac{w_m}{x_m} > \frac{w_o}{x_o} \]

5. \( m_m \) vs \( m_o \): when \( \frac{k}{\sigma^2\lambda} \) sufficiently small \( m_m - m_o > 0 \) and when sufficiently large \( m_m - m_o \) has the same sign as \( \delta \). (comment: messy to try to get characterization for the whole parameter space)
6. Principle’s welfare per-agent hired: for $\frac{k}{\sigma^2 \lambda}$ sufficiently small or sufficiently large or if $\delta$ sufficiently negative then welfare is higher when one principle hires all agents, as opposed to $n + 1$ principle-agent pairs. (comment: numerically it seems that this is always true, but analytically it is very messy to try to prove for the whole parameter space)

Proof. To be inserted. ■

comparing the equilibrium contracts between observable and non-observable contracts we get

Lemma 13

1. $y_m > y_g$
2. $x_m > x_g$ iff $\delta > \rho$
3. $a_m < a_g$
4. $\frac{y_m}{x_m} > \frac{y_g}{x_g}$
5. $m_m$ vs $m_g$: when $\frac{k}{\sigma^2 \lambda}$ sufficiently large $m_m > m_g$ iff $\delta > 0$, and when sufficiently small $m_m < m_g$. (comment: messy to try to get characterization for the whole parameter space)

6. Principle’s welfare per-agent hired: for $\frac{k}{\sigma^2 \lambda}$ sufficiently large Welfare$_m <$ Welfare$_g$, and when it is sufficiently small Welfare$_m <$ Welfare$_g$ iff $\delta < 0$;

Proof. To be inserted. ■
References


